Firm Heterogeneity and Collective Agreements: a Structural Model of Minimum Wages

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Abstract

In environments with high coverage of collective agreements, unions and employers in large firms can set the minimum wage that fits them best because they can drive small competitors out of the market. This paper develops a structural search and matching model with minimum wages and collective bargaining negotiations, which is tested on Spanish data to explain why surprisingly employers are willing to raise the minimum wage: because they gain labour market power. In a counterfactual exercise, a policymaker would be willing to promote representation of small firms to set the minimum wage level, and so reduce externalities.

Keywords: Search and Matching, Minimum Wages, Trade Unions, Collective Bargaining

JEL Codes: J51, L13, F16

1 Introduction

How do Central and Western European systems of collective bargaining affect competition in the labour market? How are different sized firms affected? How could a policy maker improve the bargaining protocol? These are the sort of questions that the present work answers.

In most European countries collective bargaining occurs at a sectoral level, with low membership to unions but high coverage of agreements; in Spain, coverage reaches to 80% of employees, of which 92% are covered by sectoral collective agreements, and 16% of workers are members of a union. The law establishes a firm size threshold of 50 workers to set up a working council. The political process to reach an agreement comprises: setting working councils in firms over 50 workers, electing representatives, and establishing negotiating tables. Arguably, the most important aspect of collective agreements (CA) is the minimum wage level, affecting between 10% to 20% of workers conditional on the sector.

Participants of this process (insiders) might use minimum wages as a tool to raise wages in order to reduce competition. This work delivers a model to study minimum wage bargaining between firms and workers, and analyses its effects on competition, market power, and the firm size distribution.

The main contribution is to bring forth how the bargaining protocol, by which the minimum wage level is decided, affects firms of different sizes and the degree of competition. Minimum wages are relevant because they redistribute profits, employment, and market power along the firm size distribution. In cases like Spain, unions and employers associations in large firms have the power to set the wage floor; by inflating the minimum wage, they have the power to expand market power and profits at the expense of small competitors.

To analyse these features, I build a search and matching model with minimum wages, on-the-job search, and two-sided heterogeneity, where I introduce unions and employer's associations in large firms that bargain for the level of the minimum wage. Then, I estimate the model using the Spanish administrative records with complete employment histories of workers of four selected sectors. Comparing four sectors within the same country grants the opportunity to contrast the level of competition and market power under the same legal framework, without resorting to less robust cross-country comparisons, where it is difficult to control for the different regulations and economic environments.

The first result is that insiders (unions and employers in large firms) are interested in raising the level of the minimum wage in an environment of intense competition. Small firms are affected by wage increases that restrict their size and number; and consequently large firms curb on rivalry. This is the case of the Trading Metal and Non-metal Industry sectors. Conversely, in an environment of mild competition, the opposite is true, which is the case of the Metal Industry and the Construction sectors.

There are three mechanisms that drive this result. First, small firms suffer an increase in costs more than large firms due to the rise in the fraction of people earning the minimum wage. Second, the minimum wage has spillover effects above where it binds and so small firms have to grant wage increases beyond the minimum. Third, small firms hire relatively more from unemployment because they are less productive and find it more difficult to attract workers from large firms; with a minimum wage in place, the opportunities to hire workers from unemployment shrink. The first and second factors imply a rise in costs, leading to a reduction in expected profits, and a closure of vacancies, whereas the third implies an obstacle to hiring. Small firms lay off more workers and hire less, becoming smaller, or being driven out of the market.

On the other hand, large firms are not as much affected by the minimum wage rise, since they pay higher wages in the first place, so the fraction of workers earning the minimum is lower than in small firms and those earning right above the minimum experience milder wage increases than those in small firms. However, now large firms have a new stock of unemployed at their disposal, making it easier to fill vacancies as a result. Moreover, workers come across less outside offers to drag their current employers into competition for their services, which makes their careers (wage increases) less dynamic, and firms do not need to grant pay rises as often. Not only workers come across fewer wage offers, but also the distribution of firms from where offers originate changes dramatically in favour of large firms. As a consequence, large firms not only reduce competition but have more profits as they pay workers less.

Turning to the bargaining protocol, a threshold of 50 workers to establish a working council sorts workers and employers into insiders and outsiders, only insiders set the level of minimum wage whose effects are felt by outsiders. The second result is that a policy maker could reduce the threshold from 50 to 0. The distinction between insiders and outsiders would vanish, reducing externalities, and increasing social welfare.

In Spain, the law establishes a firm threshold of 50 workers or more to set up a working council [1]. Workers and employers in firms with a working council (insiders) can elect their representatives to negotiate the level of the minimum wage, regardless of their membership status to a union or an association. On the other side, employees and employers in small firms, and unemployed (outsiders) are left out of this mechanism to elect representatives. Insiders are willing to implement a policy that ousts competitors from the market, and which is detrimental for the outsiders. As a result, there is room for a government policy intervention, like changing the threshold, that takes into account those affected by the decision to set a minimum wage level.

The third result is that unions hold most of the bargaining power because they are able to negotiate a minimum wage level closer to their interests, as shown in counter factual estimation. Unions are interested in raising the wage level without affecting employment, whereas employers are concerned about curbing competition without increasing costs. In any case, the minimum wage level is far from the level that workers and employers in small firms would set if they were participants of the negotiating process.

This work contributes to the literature in several ways. In the literature of trade unionisa-

[1] Depending on the sector, firms with 50 workers or more range between 1.76% and 3.62% over the total number of firms with at least one employee, which constitutes between 23% and 69% of workers. Working councils are not set up automatically once the firm hires its 50th worker, independent workers or unions need to take the initiative to set up one, and consequently between 3% and 10% of firms over 50 workers have a working council depending on the industry, which represents between 2% and 39% of workers.

tion, unions bargain on behalf of their members vis-a-vis with the company for better wages, whereas management chooses the level of employment, see Booth (1995) for a review. This framework fits better in Anglo-Saxon and Nordic countries, whereas in continental Europe negotiations are carried out at a sectoral level and on behalf of a wider base of represented beyond the union members[2]. The present work departs from that literature in two ways: the level of negotiations, and the represented workers. Within the model, negotiations are carried out at a sectoral level, not within the firm, consequently collective contracts set the minimum requirements that all agents in the market have to abide by. In a recent paper, Krusell and Rudanko (2016) assume the union bargains on behalf of the whole active workforce in the sector, employed or unemployed, which makes sense since there is no heterogeneity in their model; yet it misses the fact that there is no *a priori* reason why unions should worry about the whole workforce at a sector-wide level or even the unemployed. The second departure notices that unions and employers associations do not represent only their members but a wider base of workers who can vote. This feature makes it necessary to account for the political process.

To address these issues, I introduce a collective bargaining system with two ingredients: a firm threshold to sort workers and managers into insiders and outsiders; and unions and employers associations that bargain for the minimum wage at a sectoral level, effectively endogenising the minimum wage. Also, I consider minimum wages because it is the instrument that insiders use to affect labour market outcomes (Flinn and Mabli, 2009) (FM). Finally, I consider two-sided heterogeneity since firm heterogeneity translates into a measure of firm size, which is the variable that sorts workers and employers into insiders and outsiders; whereas worker heterogeneity accounts for ability (Cahuc, Postel-Vinay, and Robin, 2006) (CPR).

The empirical literature on CA has analysed how collective contracts and their clauses

[2]It is worth noticing that few workers are members of a union, however once a union takes the initiative to set up a working council, all workers within the firm have the right to vote for their representatives, regardless of their union status. The number of representatives for each union within the firm are counted to assign proportional representation at a higher levels, like the sector.

affect labour market outcomes using quasi-experimental data and a reduced form approach. I contribute to this literature by making explicit the mechanism and the instrument to affect labour market outcomes: minimum wages; conversely Hartog, Leuven, and Teulings (2002) consider collective agreements as a whole entity without specifying how clauses of collective contracts affect the labour market. These authors test how different levels of collective bargaining affect the level of wages, they find evidence in favour of the Calmfors hypothesis (Calmfors and Driffill, 1988). This hypothesis states that extreme levels of collective bargaining, firm or national, work best in terms of wages, inflation and unemployment, whereas sectoral agreements end up with high wages, inflation and unemployment.

The seminal paper of Cardoso and Portugal (2005) shed light on this matter by taking minimum wages into the analysis, they found out that firm adjustments are absorbed reducing the wage premium and not laying off workers, contesting the Calmfors hypothesis, they find that Western European bargaining institutions that are supposed to set rigidities on firms can coexist with low levels of unemployment. Another major advancement is Card and Cardoso (2021), in this work they thoroughly analyse collective agreements in Portugal looking at the interaction between minimum wages bargained in sectoral agreements and wage premiums paid by firms. They find that the increase of wage floors compress the wage premium. I add to them by building a structural model that makes explicit the interaction between CA negotiated by unions and employers' associations, minimum wages and labour market outcomes as explained before. In the model, on-the-job (OTJ) search offers a rationale on how wage premiums come about; and introducing unions and employers explains why minimum wages rise and how they compress wage premiums.

The paper is structured as follows. Section I outlines the continuous-time search model with minimum wages. In section II unions and employers associations are considered, they Nash-bargain to set the optimal level (for insiders) of minimum wages. Section III describes the institutional setting. Section IV presents descriptive data. Section V structural estimation is considered. Section VI results on labour market outcomes are portrayed. Section VII considers a welfare analysis, and section VIII concludes.

2 Base model and the minimum wage

I construct a search and matching model with on-the-job search, minimum wages, and two-sided heterogeneity, where workers have zero bargaining power, and hiring is costly for firms. I built on the work of Postel-Vinay and Robin (2002) in order to draw two-sided heterogeneity that allows to consider firms with different sizes and a distribution of workers with different abilities. Also, I draw wage and mobility dynamics, seemingly equal worker-firm pairs with the same ability and productivity pay different wages because they have been subjected to different histories of wage offers. The model pulls from Lise and Robin (2017) the condition for the number of vacancies, which states that firms with productivity y open new vacancies until they exhaust every profit opportunity, i.e. the value of a vacancy is zero. However, in this paper the condition is related to the number of jobs as in Lise, Meghir, and Robin (2016); both formulations are equivalent because only the steady state is considered, nonetheless steady state distributions are easier to interpret as it is clear in this section. Finally, I follow the works of Cahuc, Postel-Vinay, and Robin (2006), Flinn and Mabli (2009), and Flinn and Mullins (2019) to introduce the minimum wage level.

2.1 Setting

The economy is populated with a continuum of workers indexed by x, representing ability, which is exogenously given, publicly observable, and distributed over the interval $x \in [\underline{x}, \infty]$ according to a log-normal distribution $l(x)$ with parameters $(1, \sigma_x)$, and the total number of workers is normalised to $L = 1$. Workers are either unemployed or employed, in both cases they search for jobs to find better alternatives, the search effort of employed is s and the unemployed effort is normalised to one. Since workers search for other jobs while employed, they have the opportunity to bring a potential poaching firm into Bertrand competition with their incumbent employers in order to gain a pay rise or change jobs otherwise, the process will be explained in detailed in the following sections. Let $u(x)$ be number of workers of type x among the unemployed and $U = \int u(x)dx$ total unemployment.

The exogenous number of firms in the economy is Γ. Firms are ranked according to technology y that is distributed according to a gamma function $\Gamma(y)$ in $y \in \left[\underline{y}, \infty\right]$ with shape and scale parameters, γ_{sh} and γ_{sc}, this technology is given, and remains fixed during the whole period, i.e. no shocks to productivity. Firms with technology y hold a number of jobs $n(y)$, which are either filled or vacant, and the total number of jobs held by all firms is $N = \int n(y)dy$, both N and $n(y)$ are endogenously determined by the free-entry condition (FEC). The number of vacancies opened by the firm with productivity y is $v(y)$ and the number of workers of type x employed in this firm is denoted by $h(x, y)$, the distribution of firms across workers is then $h_y(y) = \int h(x, y)dx$, and the distribution of workers across firms is $h_x(x) = \int h(x, y)dy$. The total number of vacancies opened in the economy is $V = \int v(y)dy$ and the total number of employed people is $H = \int h_y(y)dy$.

All agents in the economy discount time at the same factor ρ. Matches are exogenously terminated by a Poisson process with parameter δ or endogenously when there is a job-to-job transition. The flow income as unemployed is $f(x, y) = xb$ whereas upon matching the firm and the worker start producing a flow output $f(x, y) = xy$, this production function exhibits a constant returns to scale on workers' abilities, so there are no complementarities among workers and they are perfectly substitutable within the firm [3].

2.2 The Matching Process

Search is random within the matching set, which under the case without minimum wages is $[\underline{x}, \infty] \times [\underline{y}, \infty]$. Unemployed and employed workers compete for vacancies with different search intensities, the search intensity of unemployed is normalised to one and for employed

[3] Other choices on the form of the production function like constant elasticity of substitution could have been considered, however the complementarities derived from them results in firms and workers being sorted. Lack of sorting is preferable as the effect that minimum wages have on sorting is clearer.

is denoted by s.

Let k be a parameter that characterises all key rates of meeting and which definition is:

$$k = \frac{M\left(U + s(L - U), V\right)}{\left[U + s\left(L - U\right)\right] V}.$$

Where M is a Cobb-Douglas meeting function of the searchers and vacancies with equal weights and a meeting efficiency parameter η. This notation is different from the canonical model (Pissarides, 2000), with the advantage that it makes explicit what the Poisson rates are composed of. Now we can define the Poisson rate at which unemployed workers meet a y-type vacancy as $kV \cdot \frac{v(y)}{V} = kv(y)$, whereas employed workers meet vacancies at a rate $skv(y)$. On the other side of the market, vacancies meet unemployed workers at a Poisson rate $ku(x)$ and meet employed ones at a rate $skh(x, y)$.

If we introduce a minimum wage m^*, the matching set is restricted above the $xy = m^*$ line. The minimum wage changes the wage and mobility dynamics, and distributions for the number of employed workers and vacancies are conditioned on x, as it is shown in the following sections. See figure 1 for reference.

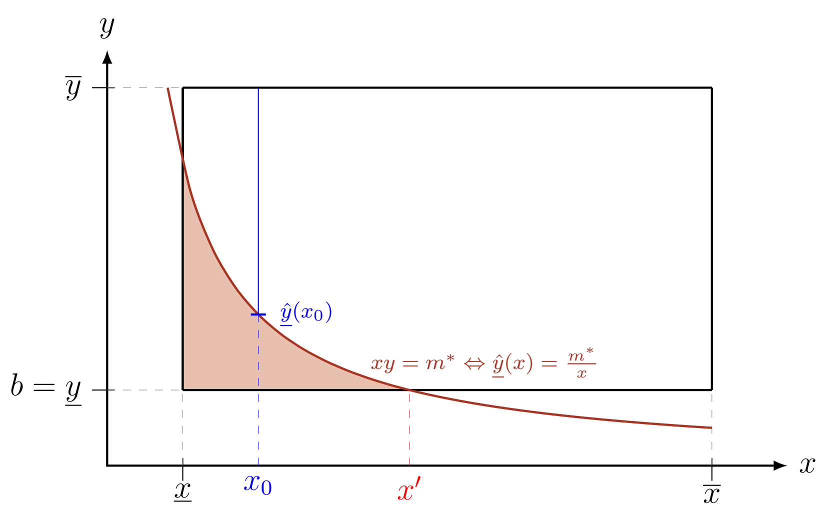


Figure 1: Matching Space

2.3 Value functions

Upon matching a worker, firms take all the surplus for themselves, which differs from Dey and Flinn (2005) and Cahuc, Postel-Vinay, and Robin (2006) where workers are assumed to hold some bargaining power. In any case, the bargaining power of unskilled workers, the case of this study, is close to zero as these authors show, and the data does not allowed to identify it without further assumptions. The only way a worker can demand a pay rise to its employer is by pulling other poaching firms into Bertrand competition. If the productivity of the poaching firm is higher than the incumbent one, the worker will move to the new firm, possibly with a pay cut as workers trade off present wages against future pay increases. The minimum wage interferes in this process by restraining the set of wage offers, which grants the worker an advantage to negotiate higher wages upon meeting a poaching firm.

2.3.1 Employed

The value of an employed worker with ability x, working in a firm with productivity y, earning the wage w is $W_1(w, x, y)$, the unemployment value is denoted $W_0(x)$, and thus the worker surplus is $W_S(w, x, y) = W_1(w, x, y) - W_0(x)$. Also, $S(x, y)$ is the surplus of the match, i.e. the surplus accounted to a worker plus the one to the firm, which is defined later. Working in terms of the surplus characterises the wage and mobility dynamics without burdening the notation.

As x-type workers in y firms search on-the-job, they can find alternative wage offers $\phi(x, y', y)$ coming from firms with productivity y'. Once an offer is found, firms compete *à la Bertrand* for the worker's services. Conditioning on the poacher's productivity, there are four possible outcomes shown below.

In the first case, the x-type worker encounters a wage offer from a firm y' that does not even have enough productivity, say $y' \leq q(w, x, y)$, to pay for the current wage and make profits, i.e. $xy' - w \leq 0$.[4] So, the worker stays with her current firm at the same wage.

[4] the threshold $q(w, x, y)$ leaves the worker indifferent between extracting the whole surplus of the poacher

In the second case, the poacher ranks in $y' \in (q, y]$, so the incumbent is more productive than the poacher, yet the latter has enough productivity to make a wage offer to the worker and make profits. Now, the incumbent firm counteroffers with a wage that leaves its worker indifferent between the two firms, retaining the worker, and granting her a pay rise.[5]

In the next two cases, the productivity of the poacher is $y' > y$, who makes a wage offer unmatchable for the incumbent that cannot retain the worker. Since the surplus value of the x-type worker is increasing in w and y', the poacher can trade off lower wages in the present for higher wage increases in the future. However, not every pair (y, y') is able to play a "wage war" because there is a binding minimum wage; in particular, when the poacher is "much" more productive, say $y' \geq t(x, y)$, than the incumbent, the former will not be able to lower the wage in its full extend to the worker.[6] Hence, the worker has the opportunity to work at high productive firms earning no less than the minimum, effectively rising the value of employment.

From the previous discussion, the surplus continuation value of a worker net of lay-off shocks, and future wage offers in the presence of the minimum wage would be

$$\begin{aligned}\left(r + \delta + sk\overline{V}(q)\right) W_S\left(w, x, y\right) = w - rW_0(x) + sk \int_{q(w,x,y)}^{y} S(x, y')v(y')dy' \\ + sk \int_{y}^{t(x,y)} S(x, y)v(y')dy' + sk \int_{t(x,y)}^{\overline{y}} W_S(m^*, x, y')v(y')dy'.\end{aligned}$$

Where $\overline{V}(y) = V - V(y)$ and $V(y)$ is the amount of vacancies coming from firms with productivity y or less. The first term in the right-hand side is the flow wage earned by the worker. The second is the continuation value of the loss when the match is destroyed. The third one accounts for the increase in value to the worker when promoted to a higher wage. The fourth is the continuation value that comes from a job-to-job transition, and the last

or staying in her current firm earning the same wage, i.e. $W_S\left(w, x, y\right) = S(x, q)$.

[5]Formally, let $\phi(x, y', y)$ be the offer done by firm $y' < y$ to an x-type employee working at firm y and which is implicitly defined as $W_S(\phi(x, y', y), x, y) = S(x, y')$.

[6]The wage offer $\phi(x, y, y')$ is defined as $W_S\left(\phi(x, y, y'), x, y'\right) = S(x, y)$ when the productivity of the firm is $y' \in (y, t(x, y)]$. And $\phi(x, y, y') = m^*$ when the firm ranks in $y' \in (t(x, y), \infty)$. The threshold $t(x, y)$ is again implicitly defined as $W_S\left(m^*, x, t(x, y)\right) = S(x, y)$

term is the increase in expected value due to m^*. Notice that the worker does not need to earn the minimum wage to increase her surplus value. As a result, workers will demand higher wages in the negotiations, increasing their wages beyond the minimum. However, for employed workers mobility dynamics are not affected by m^*. For derivations of the value functions, and wage expressions along with their computational implementation, see appendices A, and B respectively.

2.3.2 Employed earning the minimum wage

Also, we have to consider what is the value for an employed worker earning the minimum wage. Since there is a minimum wage in place, low productive matches, $xy < m^*$, are not viable any more, and the firm with the minimum viable productivity to hire a x-worker is restricted to $\underline{\hat{y}}(x) = \frac{m^*}{x}$. At the minimum wage the value function is

$$\left(r + \delta + sk\overline{V}(q)\right) W_S\left(m^*, x, y\right) = m^* - rW_0(x) + sk\int_{\hat{y}(x)}^{y} S(x, y')v(y')dy' + sk\int_{y}^{t(x,y)} S(x, y)v(y')dy' + sk\int_{t(x,y)}^{\overline{y}} W_S(m^*, x, y')v(y')dy'$$

The only thing that is likely to change is the lower limit of the integral in the third term. Now, any firm above $\hat{y}(x)$ can make a wage offer and earn profits, as the productivity of the match is higher than the minimum wage.

2.3.3 Unemployed

The value of unemployed worker of type x is denoted by $W_0(x)$ and receives a flow bx while unemployed, which has to be given up when unemployed. The term b is common to all workers, and is the minimum productivity that a firm needs to attract a worker from unemployment. In addition, The flow depends on the ability x, workers with high-skill have better wages while employed and enjoy higher benefits as unemployed. The wage offer encounter by an unemployed worker is $\phi_0(x, y)$, which depends on x and y.

The minimum wage affects the mobility dynamics of the unemployed, in which we have to consider two regions, see figure 1. From $[\underline{x}, x']$ matches below the line $xy = m^*$ are not longer possible and the set of available firms is restricted above $\hat{\underline{y}}(x) = \frac{m^*}{x}$. From $[x', \infty]$, mobility dynamics are not affected. From these considerations the following lemma states:

LEMMA 1. *The minimum viable productivity of a firm $\hat{\underline{y}}(x)$ to hire a worker under the presence of a wage floor is:*

$$\hat{\underline{y}}(x) = \begin{cases} \frac{m^*}{x} & \text{if } x < x' \\ \underline{y} = b & \text{if } x \geq x' \end{cases}$$

Proof. *See appendix C.1.*

A consequence of this lemma is that as $m^* \to 0$, $\hat{\underline{y}}(x) \to \underline{y} = b$, in the whole domain of x.

Under the minimum wage the unemployment value increases because firms with high enough productivity cannot trade off less entry wages today for pay rises tomorrow. From $[\underline{x}, x']$, the unemployment value is equivalent to working in the firm with the minimum viable productivity $\hat{\underline{y}}(x)$ at a wage $\phi_0(x, \hat{\underline{y}}(x)) = m^*$, which is the minimum wage. The firm $\hat{\underline{y}}(x)$ cannot make surplus out the match, otherwise a firm with marginally less productivity could enter the market and make profits, a contradiction. From $[x', \infty]$, $W_0(x)$ is equal to $W_1(\phi_0\left(x, \underline{y}(x)\right), x, \underline{y})$. From these considerations the next lemma states that

LEMMA 2. *In $x \in [\underline{x}, x']$, the value as unemployed is equal to the value of first employment at $\hat{\underline{y}}(x)$. Similarly, the surplus value of the match is zero. Therefore,*

$$W_1\left(\phi_0\left(x, \hat{\underline{y}}(x)\right), x, \hat{\underline{y}}(x)\right) = W_0(x) \iff W_S\left(\phi_0\left(x, \hat{\underline{y}}(x)\right), x, \hat{\underline{y}}(x)\right) = 0$$

and

$$S\left(x, \hat{\underline{y}}(x)\right) = 0$$

In $x \in (x', \infty)$, the value as unemployed is equal to the value of first employment at $\underline{y}$.

Therefore,

$$W_1\left(\phi_0\left(x,\underline{y}\right),x,\underline{y}\right)=W_0(x) \iff W_S\left(\phi_0\left(x,\underline{y}\right),x,\underline{y}\right)=0$$

Proof. See appendix C.2.

Finally, we need to work out the entry wage under the presence of the minimum wage. In the range $[\underline{x},x']$ the entry wage is the minimum wage m^*, clearly $\underline{\hat{y}}(x)=\frac{m^*}{x}$, which is only able to offer no more than the minimum wage. Firms with higher productivity than $\underline{\hat{y}}(x)$ would but cannot undercut below m^*. For abilities in $[x',\infty]$, entry wages follow the same dynamics as when a worker employed at $\underline{y}=b$, and receives an offer form a firm $y'>b$, as shown in previous sections. For key derivations and proofs, see appendix B.

2.3.4 Vacant Jobs

Since only the steady state is considered and the distribution of vacancies and filled jobs remain constant, choosing the number of vacancies is equivalent as choosing the number of jobs to hold. Because we are considering individual firms, we need to define the average number of jobs held by the y-type firm, which is $\frac{n(y)}{\Gamma\gamma(y)}$. So, firms create jobs at flow cost $c\left(\frac{n(y)}{\Gamma\gamma(y)}\right)$ until the marginal cost of opening a vacancy $c'\left(\frac{n(y)}{\Gamma\gamma(y)}\right)$ equals the expecting value of filling it. Define the continuation value of holding a vacancy under the presence of the minimum wage as

$$\rho\Pi_0(y)=-c'\left(\frac{n(y)}{\Gamma\gamma(y)}\right)+kJ(y),$$

where

$$J(y)=\int_{\underline{x}}^{\overline{x}}S(x',y)u(x')dx'+s\int_{\underline{x}}^{\overline{x}}\int_{\underline{\hat{y}}(x)}^{y}\left(S(x',y)-S(x',y')\right)h(x',y')dy'dx'.$$

The first term in the right-hand side is the marginal cost of filling a vacancy, and the second term, $kJ(y)$ is the expected value of filling a vacancy, which is comprised of: the expected value of hiring a worker from unemployment in the first term, and the expected value of hiring an employed worker from a firm with lesser productivity in the second term. Notice that to work out an expression when $m = 0$, we just need to replace $\hat{\underline{y}}(x)$ by $\underline{y}$ in the second integral.

2.3.5 Surplus

Let us determine the continuation surplus of holding a filled job as the difference between its continuation value and the value of a vacancy, i.e. $\Pi_S(w, x, y) = \Pi_1(w, x, y) - \Pi_0(y)$. The continuation surplus is then defined as $S(x, y) = \Pi_1(w, x, y) - \Pi_0(y) + W_1(w, x, y) - W_0(x)$.

Under the minimum wage, the surplus is affected in two ways. First, by increasing value of unemployment conditioned on participating in the labour market. Second, by the increase in value of an employed worker who expects to work in high productive firms with a higher salary; the value of a job $\Pi_S(w, x, y)$ is not affected by this fact, since the y-type firm always expects zero profits when its x-type worker quits to a more productive one $y' < t(x, y)$. Then the expression of the surplus under the minimum wage is

$$\left(r + \delta + sk\overline{V}(t_{(x,y)})\right) S(x, y) = yx - rW_0(x) + sk \int_{t(x,y)}^{\overline{y}} W_S(m^*, x, y')v(y')dy'.$$

2.4 Equilibrium

The exogenous elements of the model are the distribution of workers $l(x)$, with support in $[\underline{x}, \infty]$, $\Gamma(y)$, the support of this distribution,$[\underline{y}, \infty]$, the discount factor ρ, the job destruction δ, the search intensity of employed workers s, the value of leisure b and the production technology $f(x, y) = xy$. Given these parameters, the equilibrium can be characterised by defining the distributions of employees, unemployed, vacancies, and wages, along with the free entry condition.

2.4.1 Balance Equations

In equilibrium, the distribution of the unemployment rate of workers of type x, $u(x)$, and the number of vacancies of type y, $v(y)$, is determined according to the balance conditions

$$\int h(x,y)dy + u(x) = l(x)$$
$$\int h(x,y)dx + v(y) = n(y).$$

The first condition states that the number of employed workers plus the number of unemployed is equal to the number of people of ability x. The second says that the number of filled jobs plus the number of vacancies is equal to the number of jobs with productivity y.

2.4.2 Flow Equations

The distribution of wages, matches, unemployed, and vacancies, i.e.: $G(w|x,y)$, $h(x,y)$,$u(x)$, and $v(y)$ respectively, follow a steady state flow equation where inflows balance outflows, and which expressions are:

$$u(x) = \frac{\delta}{\delta + kV} l(x)$$
$$v(y) = \frac{\delta + sk\overline{V}(y)}{\left(\delta + sk\overline{V}(y)\right) + (kU + skH_y(y))} \cdot n(y)$$
$$h(x,y) = \begin{cases} \frac{1}{H} h_y(y) h_x(x) & if \quad xy \geq m^* \\ 0 & if \quad xy < m^* \end{cases}$$
$$G(w|y) = \begin{cases} \frac{h_y(q)}{v_y(q)} \cdot \frac{v_y(y)}{h_y(y)} & if \quad xy \geq m^* \\ G_0 & if \quad xy = m^* \text{ or } \underline{y} = b \\ 0 & if \quad xy < m^* \end{cases}$$

Where G_0 is the mass point in the distribution of wages in the lower support, and $h_y(y)$ depends solely on $n(y)$, which at this point is exogenously determined. The main advantage of having introduced firm heterogeneity is that we have a measure of firm size in one-to-one correspondence with productivity. This fact turns out to be an essential ingredient for the political economy, where unions and employers in large frims bargain. Finally, the number of jobs created by firms of type y, $n(y)$, is set by the free entry condition described below. For derivations of key equations without and with minimum wages see appendices D.1 and D.2 respectively.

2.4.3 Free Entry Condition

Firms of type y exert increasing effort in recruiting candidates until the cost of maintaining a vacancy and retaining talent equals the expected value of filling it for every y, $\Pi_0(y) = 0$, at equilibrium:

$$c'\left(\frac{n(y)}{\Gamma\gamma(y)}\right) = kJ(y). \tag{1}$$

Following the parametrisation of Lise and Robin (2017), the cost of filling a vacancy is defined as

$$c\left(\frac{n(y)}{\Gamma\gamma(y)}\right) = c_0\frac{\left(\frac{n(y)}{\Gamma\gamma(y)}\right)^{1+c_1}}{1+c_1}. \tag{2}$$

Then, deriving 2, inserting it in 1, and solving for $n(y)$, the equilibrium number of jobs by y-type firms is

$$n(y) = \left(k\frac{J(y)}{c_0}\right)^{\frac{1}{c_1}}\Gamma\gamma(y).$$

Finally, the aggregate number of jobs in the economy is worked out by summing over all companies $N = \int n(y')dy'$.

3 Political Economy

The chief contribution of this work is to endogenise the minimum wage level by reckoning the role of unions and employers. In collective bargaining systems where the bulk of negotiations are carried out at a sectoral level, what is agreed between unions and employers is usually extended to other participants in the labour market. I focus on the extreme case where collective agreements are applied to the whole labour market, regardless of workers and firms being members to their representative associations, or having participated in the political process.

There are three stages to consider in the political process. First, who is allowed or able to set a vote? Either because legal clauses, or collective action constraints, participation in the negotiating table is subject to firms reaching a certain size, $\overline{h}$. Only workers in these firms vote, and are heard, regardless of their union membership status. Because in the model there is a strictly increasing relation between size and productivity, setting a minimum size threshold is equivalent to define the firm with minimum productivity above which workers can vote $\tilde{y}$, as:

$$h_y(y) \geq \overline{h} \Leftrightarrow y \geq \tilde{y} = h_y^{-1}\left(\overline{h}\right).$$

The second stage relates to union and employers preferences. In the present paper I deviate from the assumption that unions and employers' associations represent their members, and instead they appeal to workers who can vote. The utility of the union is that of an utilitarian objective function like

$$T(m) = \int_{\tilde{y}}^{\overline{y}} \int_{\underline{x}}^{\overline{x}} \int_{m}^{\overline{w}} \left(W_1\left(w, x, y\right) - W_0\left(x\right)\right) G(w|x, y) h(x, y) dw dx dy.$$

And same is applicable in the firm side

$$E(m) = \int_{\tilde{y}}^{\overline{y}} \int_{\underline{x}}^{\overline{x}} \int_{m}^{\overline{w}} \left(\Pi_1 \left(w, x, y\right) - \Pi_0 \left(x\right)\right) G(w|x, y) h(x, y) dw dx dy.$$

Considering partial equilibrium effects, unions face the typical trade off, higher wages despite higher unemployment for their represented; all firms are worse off because they pay higher wages; in addition, low productive firms are not able to hire low productive workers. Accounting for general equilibrium effects, less employment means vacancies are easier to fill, especially for those firms that do not have to lay off workers; also, high skill workers encounter wage offers more frequently. The net effect is ambiguous and structural estimation is carried out to discern what effect is stronger.

In the third stage, unions and employers set the level of the minimum wage that maximises their respective objective functions weighted by their bargaining power $\alpha \in [0, 1]$, conditional on being in large firms. Formally, they maximise the Nash bargaining solution, in which I take an axiomatic approach and make no assumption about α. Nonetheless, it might be reinterpreted as the relative discounting factors (Rubinstein, 1982). Then,

$$m^* = \arg\max_{m} \quad E^{1-\alpha}(m) \cdot T^{\alpha}(m). \tag{3}$$

From this specification, the utility of trade unions and employers can be estimated from data on wages, transitions, and firm size; and m^* is readily available; whereas α is the only parameter that cannot be observed. However, setting the FOC with respect to m in the bargaining programme 3, we can work out an expression for α as a function of $E(m^*)$, $T(m^*)$, and m^*:

$$\alpha = \frac{\frac{E'(m^*)}{E(m^*)}}{\frac{E'(m^*)}{E(m^*)} - \frac{T'(m^*)}{T(m^*)}}. \tag{4}$$

where $E'(m) = \frac{\partial E}{\partial m}$, and $T(m) = \frac{\partial T}{\partial m}$. This expression will turnout to be essential when we simulate counterfactuals for different firm size thresholds.

4 The Institutional Setting and Data

4.1 Institutional setting

Spain is framed within the class of collective negotiating systems where bargaining is a public good (or bad), typical of continental Europe. These systems are characterised by a high coverage of CA but low membership to unions. Nonetheless, Spain has some peculiarities.

First, the important agent is the works council, which is the collegiate organism within companies of more than 50 workers in charge of representing the staff. This council is composed of 5 to 75 members, depending on the size of the firm, and is directly elected among the whole workforce, regardless of their union membership. However, workers have to launch a works council, since it is not automatically started. Conditional on the sector, between 3% and 10% of firms have a works council, which represent between 2% and 38% of the employed workers.

Another feature is that the key institution at a sectoral level is the 'Bargaining Commission', which is the body in charge of reaching an understanding, and is composed of employers associations and trade unions. The ministry of labour records the union status of elected committee members in each firm, and assigns representation in the bargaining commission to different unions in proportion to the number of delegates at a firm level.

The last feature is that the 'Collective Agreement' at a sector level rules over the whole sector on any topic related to labour. The agreement is published in the Official Bulletin of the State (BOE), has rank of law, and Judges use it to solve disputes. Then, this collective contract serves as the minimum standard individual contracts must have.

4.2 Employment Histories

To test the model described in previous sections I use the *Muestra Continua de Vidas Laborales* (MCVL), a database with complete and exhaustive information on working histories of employees. The MCVL is a 4% sample of population having a relation with the General Treasury of the Social Security (TGSS) in the years of reference (2015-2017). The TGSS is the institution in charge of the social security finances and releases this database on a yearly basis.

One observation is a spell with information about the employment history of the worker, the monthly base wage, and the firm size. the sub-sample of workers selected for the present study are those in four collective agreements with enough data to carry out the estimations, see the last four rows of table 1. As it is clear, they have enough observations to identify UE, EU, and EE transitions, and wage and size distributions. Having four different sectors grants the advantage to compare the firm size, the minimum wage, and the level of competition within the same legal framework.

Table 1: Number of observations and means for selected variables

Collective agreement/ Industry	Number of workers	Number of spells	Number of employment spells	Duration of employment spells	Number of unemployment spells	Duration of unemployment spells	Number of firms	Size (mean)
Oil	8	8	4	68.8	4	49.3	4	12.0
Coffee	139	140	90	88.8	50	25.5	49	80.9
Globes	167	180	103	82.6	77	20.4	74	22.7
Plastics	137	142	116	116.6	26	24.3	62	42.1
Sewage	105	107	86	55.0	21	23.1	19	1789.9
Street Cleaning	12	12	10	93.3	2	6.5	8	14.7
Pastries	282	294	190	78.3	104	35.6	114	70.1
Wood industry	91	95	72	114.3	23	17.2	52	16.7
Metal trading	3149	3251	2175	92.7	1076	25.9	1517	46.5
Metal industry	3710	3871	2859	98.1	1012	27.9	1743	115.3
Non-metal industry	2369	2467	2004	101.3	463	36.2	1039	175.7
Construction	5346	5671	2983	66.9	2688	16.5	2228	44.0

Descriptive variables for several CAs in the region of Madrid taken from the employment histories database. Durations are mean durations measured in months, and the variable size is the mean size of a firm in that industry

Workers in these four collective agreements are grouped into seven categories according to wage floors, in this paper, the lowest wage floor is considered, as it act as the minimum wage for that industry. Considering several wage floors is out of the scope on two grounds. First, there is not enough data to identify transitions and distributions for each category. Secondly, to bring into the model the occupational ladder is an exciting and non-trivial challenge, as one should also take into account the role of experience to account for movements in occupations, two excellence references in this respect are Burdett, Carrillo-Tudela, and Coles (2011) and Bagger et al. (2014).

Data on wages is in full-time equivalent units (8 hours per day). Although Katz and Krueger (1992) find that a wage increase causes firms to substitute part-time jobs by full-time employment, the intensive margin is left out of the analysis since the model is not well suited for purpose. Administrative records only track periods of formal employment, so non-employment periods are considered and labour force participation is out of the scope. Finally, workers with a salary of more than 5000 euros per month have been removed from the sample, as they distort the wage distribution unreliably.

4.3 Collective Agreements

The database has accurate and thoroughly information about CAs at firm and sectoral levels. Interestingly, the REGCON has information about the number of employees covered by the CA, and the number of representatives at a firm level. The Ministry of Labour counts union representatives to assign representation at a higher levels, e.g. sectoral level, in the negotiating table. Large firms usually have working councils whereas small firms do not, then, large firms tend to be overrepresented in the decision making process.

The model is muted about over(under) representation in the negotiating table, but it does a clear distinction of those who are in and out depending on the firm size. A compromise to bridge the model with reality is to set the cut-off at the firm size of the median representative distribution, which critically comes from the REGCON database. Firm size cut-offs for the four CA are in table 2.

Table 2: Firm size cut-offs for the four CA

	Metal industry	**Non-metal industry**	**Trading metal**	**Construction**
Firm size cut-off (workers)	274	369	164	202

As I focus in four CAs in the province of Madrid, the collective agreements that apply are clearly identified. Nonetheless, the skill categories that the social security assigns to workers does not have to coincide with the ones negotiated in CAs (Adamopoulou and Villanueva, 2020). So, I have matched skill categories for the MCVL.

Card and Cardoso (2021) do a vast linkage of CAs to worker level data, this process turns out to be insurmountable as the TGSS does not deliver such information. Although the MCVL has two advantages with the respect to *Quadros de Pesoal*, the Portuguese counterpart. First, CAs are usually negotiated for long periods that overlap with the period of validity of the agreement, consequently retroactive measures have to be taken. The advantage of MCVL is that it writes a correction in the months affected, thus backdating is not a worry.

Another concerned could have been the statuary minimum wage which potentially could overlap with wage floors considered. This is certainly not the case, as the wage floors of the four sectors are well above the national minimum wage at the time considered (2015-2017).

5 Estimation

5.1 Method

Using the data described in the previous section, I estimate the model using the classical minimum distance (CMD) estimation (Newey and McFadden, 1994; Wooldridge, 2010). Details of its implementation are in appendix E.

At this point, the main concern is what moments to choose to identify the vector of parameters $\boldsymbol{\theta} = \left\{\underline{x}, \sigma_x, s, \eta, \underline{y}, \gamma_{sh}, \gamma_{sc}, c_0, c_1\right\}$. This work shows a heuristic representation of identification as in the works of Lise and Robin (2017) and Lise, Meghir, and Robin (2016). I use the observed mean wage, wage variance, and the mass point at the minimum wage in order to identify parameters related to worker ability $\underline{x}, \sigma_x$ and partially firm productivity $\underline{y}, \gamma_{sh}$ and γ_{sc}. The higher the variance of worker ability and the parameters of firm productivity the larger the observed upper support of the wage distribution, and mean and variance will increase accordingly. Depending on how concentrated is the wage distribution, σ_x γ_{sh}, and γ_{sh}, skew wages towards the left or right tail depending on the relative strength of the two parameters.

Moments related to durations serve to identify the relative search intensity s and matching efficiency η, the more effort employed workers exert with respect to those unemployed, the more alternative firms they encounter that allows them to experience a job-to-job transition more often which results in shorter tenures, so job duration will serve as a moment to identify search intensity. Since s is a relative search effort with respect to those unemployed, unemployment durations are taken to identify the parameter. Last but not least, the employed-to-unemployed ratio is used to identify parameters related to the matching function.

Deciles of the size distribution of firms are intimately related to the firm productivity, i.e. $\underline{y}, \gamma_{sh}$ and γ_{sc}, and the costs of opening vacancies c_0 and c_1, which are linked to the free entry condition. The lesser the costs the more vacancies each company opens, there will be more matches and consequently k will rise and firms will become lager, in turn durations will be affected accordingly by this.

5.2 Estimation of Parameters

Table of the full set of estimated parameters for the four sectors is presented in table 3.

The minimum ability of workers ranks between $[1.38, 1.639]$, and the variance of ability σ_x between $[0.56, 0.68]$, meaning that the distribution is right-skewed with most workers

Table 3: Parameters

	Metal industry	**Non-metal industry**	**Trading metal**	**Construction**
Worker abilities				
$\underline{x}$	1.38	1.639	1.483	1.454
σ_x	0.68	0.554	0.595	0.601
Matching				
s	1.89	1.80	1.871	2.082
η	0.08	0.09	0.052	0.062
Firm				
$\underline{y}$	260.77	278.03	260.98	229.16
γ_{sh}	17.72	16.78	15.64	16.29
γ_{sc}	12.74	13.08	13.74	14.09
Vacancy costs				
c_0	28.19	33.67	33.64	32.57
c_1	2.02	2.17	1.93	1.87

concentrated in a relative low ability interval, as expected because low-skill workers were considered. Search effort exerted by employed workers is between [1.8,2.1]. Parameters that govern the number of jobs, c_0 and c_1, are in $[28.2, 33.7]$ and $[1.9, 2.2]$ respectively. The baseline parameter c_0 is very low, it costs very little to open a single vacancy, whereas c_1, which governs convexity, is high, firms find very costly to open additional vacancies, curtailing the number of large firms, which explains the few number of large firms encounter in the data; for example, to hire 3 workers would cost €258 in the metal industry, whereas to hire 10 workers would cost €9,774, and so productivity has to be very large in other to encounter large firms. Turning to firm related parameters, $\underline{y}$ is relatively low, and γ_{sh} and γ_{sc}, that govern the shape of the distribution imply that it is slightly right-skewed.

5.3 Fit

Table 4 confronts moments taken from data against those implied by the model at the optimal $\hat{\theta}$. Overall, the fit is very good. In relation to wages, moments coming from the model mimic closely their real counterparts, with the exception of the percentage of workers earning the minimum wage in the metal industry. Moments related to duration perform well, unemployment duration fits real data closely, whereas those of tenure tend to be underestimated by the model. In connection with the previous point, there are many large firms that compete for workers, in turn, workers find it relatively easy to find new opportunities to transit to higher paying firms. Now, turning to distribution of firms across workers we see that the model underestimates the number of workers in low productive firms, whereas it has a thick long right-tail. High productive firms always win the sequential auction model of offers and counter offers. As such, in order to generate a wage distribution with a big mass point at the minimum, large firms have to be encounter.

Table 4: Fit of the data

	Metal industry		**Non-metal industry**		**Trading metal**		**Construction**	
	real	model	real	model	real	model	real	model
% at MW	0.15	0.26	0.24	0.20	0.10	0.13	0.16	0.16
Mean log-wage	7.44	7.51	7.42	7.52	7.45	7.34	7.32	7.41
s.d. log-wage	0.38	0.37	0.41	0.34	0.43	0.35	0.37	0.33
Unempl. duration	16.23	19.16	6.89	6.45	13.75	10.85	20.88	19.03
Tenure duration	52.53	38.2	53.19	31.79	50.79	30.9	43.8	25.21
Number of spells	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.03
Size perc 1	1.10	0.45	1.61	0.50	1.10	0.37	0.69	0.33
Size perc 2	1.79	1.10	2.20	1.23	1.61	0.85	1.39	0.73
Size perc 3	2.30	1.80	2.71	2.00	2.08	1.41	1.79	1.21
Size perc 4	2.71	2.47	3.04	2.70	2.40	1.99	2.08	1.71
Size perc 5	3.09	3.11	3.50	3.34	2.77	2.58	2.48	2.22
Size perc 6	3.50	3.75	3.83	3.97	3.18	3.17	2.83	2.76
Size perc 7	4.01	4.48	4.41	4.65	3.64	3.80	3.22	3.36
Size perc 8	4.50	5.42	5.05	5.51	4.13	4.58	3.76	4.10
Size perc 9	5.42	6.73	5.82	6.84	4.84	5.80	4.56	5.25

6 Welfare and Bargaining

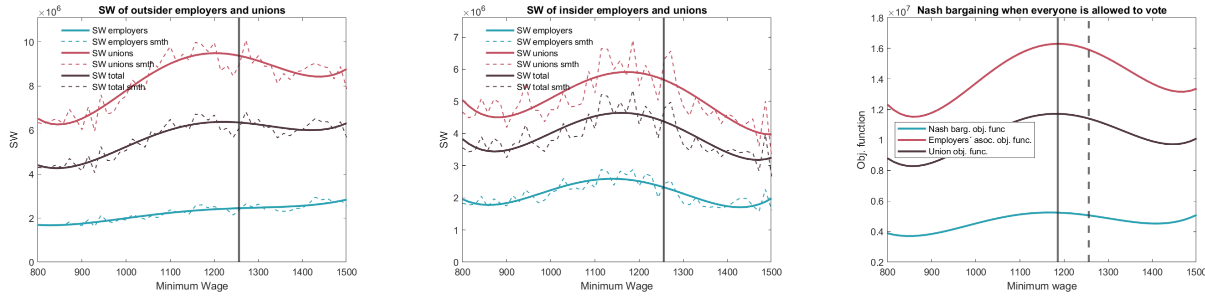
Before analysing the welfare implications of the search and matching model, along with the bargaining system outlined in sections 2 and 3, it is necessary to retrieve the social welfare functions (SWF) of unions and employers associations, which is done in three stages. First, the value functions of workers and employers are retrieved at the observed level of the minimum wage and at $\widehat{\boldsymbol{\theta}}$. Secondly, using information about the firm size cut-off to assign representation in the negotiating table, which is left again in table 5 for reference, I estimate the value functions of unions and employer's associations. Thus, m^* is observed, and $\widehat{E(m^*)}$, $\widehat{E'(m^*)}$, $\widehat{T(m^*)}$ and $\widehat{T'(m^*)}$ are obtained from the model. Third, we solve equation 4 numerically from model estimations to predict bargaining power of unions ($\hat{\alpha}$) in the different CAs, which are in table 5.

Table 5: Firm size cut-offs for the four CA

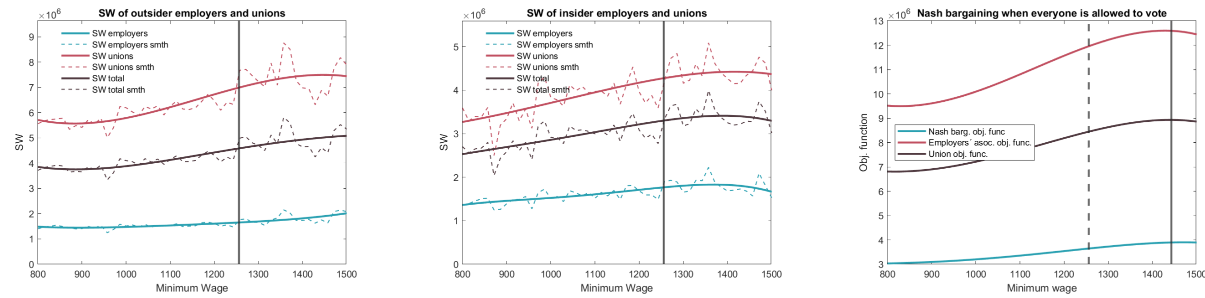
	Metal industry	**Non-metal industry**	**Trading metal**	**Construction**
$\hat{\alpha}$	0.71	0.71	0.67	0.67
m^*	1255.9	1255.9	1130.8	1290
Firm size cut-off (workers)	274	369	164	202

Figure 2 presents the SWF of unions and employer's associations for the four industries, one for each row numbered with letters. The left-hand side panels show SWF of outsiders. Unions in small firms are willing to rise the level of minimum wage, because workers are granted wage increases despite the higher probability of being unemployed. For the Metal Industry, workers would be better off with a lower minimum wage, since the losses of unemployment outweigh wage increases.

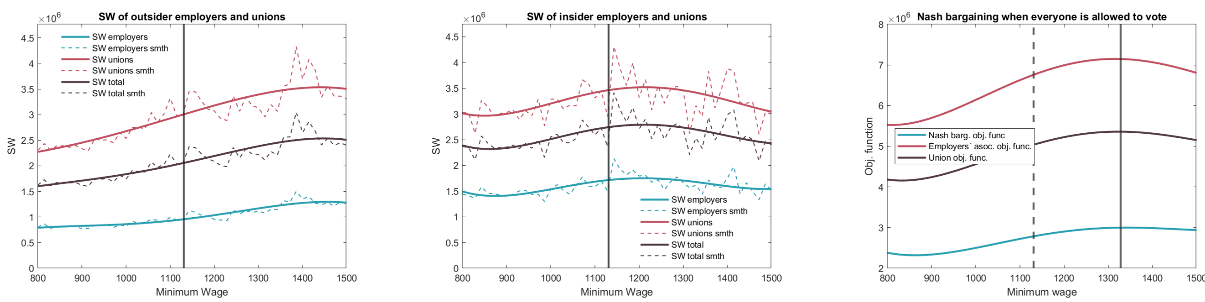
Turning to insiders, the panels on the middle, the picture is more mixed. Insider workers in the Metal Industry and Construction sectors are interested in lowering the level of minimum wages. Wage distribution in large firms stochastically dominates that of small firms, workers in large firms are not as affected by the minimum wage as those in small ones, but they still suffer from the increasing prospects of being unemployed. Firms in these industries are also interested in lowering the level of minimum wages as it interferes with their hiring process. They face the trade off of less competition against earning less from lay-offs and increasing wages. This is not the case of the Non-metal Industry and the Metal trading sectors, unions and employer's associations would be willing to rise the level of minimum wages as they remove competence from smaller firms.



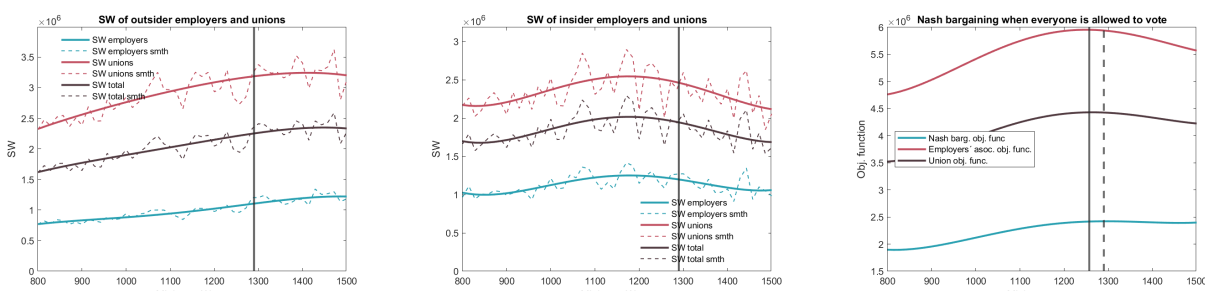
(a) Metal industry



(b) Non-metal industry



(c) Metal trading



(d) Construction

Figure 2: Left panel: objective function of outsider workers and firms, solid line current minimum wage; middle panel: objective function of insider workers and firms, solid line current minimum wage; right panel: objective function of workers and firms when everyone is allowed to participate of the bargaining process, dashed line current minimum wage, solid line new minimum wage. In dark-brown, a weighted average of the objective function of workers and firms.

Ideally, the minimum wage and the locus where the weighted objective function attains a maximum would coincide. However, there are several simplifications and challenges that need taking into account. First, it is the issue of the cut-off of the firm size, even though a natural choice is to pick the firm size of the median representative of each industry, other more refined possibilities are also available, like to give every firm a weight with the relative representation seen in the data, and effectively removing the cut-off. Another issue is the aggregation of preferences, here I assume a utilitarian SWF, which assigns every individual equal weight, nonetheless other methods can be considered.

In the right-hand side panel, we look at the counter factual analysis, where the cut-off threshold is removed and all employees are allowed to participate of the negotiating process, maybe because legal requirements. The dashed line indicates the old level of minimum wages whereas the solid line is the new level agreed by unions and employers. In the Metal Industry and Construction sectors, workers and employers associations would be willing to lower the minimum wage level by 5.2% and 3.1% respectively. In the case of the Metal Industry, it is reasonable that the level of minimum wages since both outsiders and insiders are better off with a lower level. The case of the Construction sector is more interesting, here there is a clear conflict of interests, outsiders are willing to rise the wage floor whereas insiders would be willing to lower it, when the cut-off is removed, the level of minimum wage chosen would increase, workers and small firms that were previously outsiders now can influence the political process and rise the wage floor. Looking at Non-metal Industry and Metal Trading the picture is clear, both unions and employer's associations would increase the level of minimum wage by 13.1% and 15.1% respectively. For the case of Non-metal industry, both outsiders and insider reach the maximum at the same level of minimum wages and they would be willing to rise it. Last, the Metal Trading tells a somewhat similar story, but whereas insiders would like to have a mild increase of the minimum wage, outsiders bet for strong increase. In the final picture, the old outsiders out weight the insiders and the level of minimum wages is closer to their interest.

7 Labour Market Outcomes

The rising level of the minimum wage affects labour market outcomes through different channels. On the supply side, workers who previously earned less than the minimum experience a wage rise. Simultaneously, the minimum wage has spill-overs through the wage distribution, so workers earning close to the minimum also experience a wage rise (Autor, Manning, and Smith, 2016). But, Increasing payrolls comes at cost of reducing employment. Even thought there is little evidence on disemployment effects of the minimum wage (Card and Krueger, 2015), the model presented here suggests significant reductions in hours and employability for low-skill workers (Neumark, Schweitzer, and Wascher, 2004).

On the demand side, there are three channels that interfere with the ability of firms to retain and hire workers, these channels affect differently to small and large firms. Firstly, the proportion of workers earning the minimum wage in small firms is bigger than in large ones. low-skilled workers in small firms do not produce enough to pay for the minimum wage and they are dismissed; For those who remain, the firm makes less profits. Secondly, the minimum wage has spillover effects through the wage distribution; firms cannot trade off smaller than the minimum wage today for higher wages in the future, as a result workers expect higher wages on average, and demand their employers wage increases despite not being bound by the minimum wage. Third, small firms hire mostly unemployed workers because they find it difficult to poach from other more productive firms, actually the firm with the lowest productivity can only hire from unemployment, consequently the rate at which they hire workers is lower. Thus small firms find it more difficult to hire and retain workers, posting less vacancies and becoming smaller.

On the contrary, large firms pay a lower proportion of their workers the minimum wage or close to it, so large firms do not suffer so much as small firms the increase in costs. However, large firms have a new stock of unemployed at their disposal and find it easier to fill vacancies. There is also a general equilibrium effect, large firms face less competition from smaller ones, workers come across less outside offers, and firms do not have to grant pay rises as often, increasing their profits.

In the following subsections variables like unemployment, employability, wage dynamics, spill-overs and wage inequality are analysed.

7.1 Unemployment

This section shows a counterfactual analysis for the four markets under study, showing that moderate increases of minimum wages do not have an effect on unemployment. However, sizeable increases lead to a surge in unemployment, specially among low-skill workers. Figure 3 shows unemployment rates for different levels of the minimum wage given $\widehat{\boldsymbol{\theta}}$.

(a) Metal Industry and Non-metal Industry

(b) Metal trading and Construction

Figure 3: Unemployment rate as a function of the minimum wage for the four sectors. The solid line is the current minimum wage and the dashed line is the minimum wage under the new policy

As in figure 2, dotted lines represent the current level of minimum wages, whereas the solid line represent the minimum wage agreed in the absence of a cut-off threshold. The first thing to notice is that minimum wages do not interfere with unemployment up to moderate levels that range between $m \in [800, 1100]$ approximately, points where higher level of minimum of minimum wages start causing disemployment effects depending on the sector.

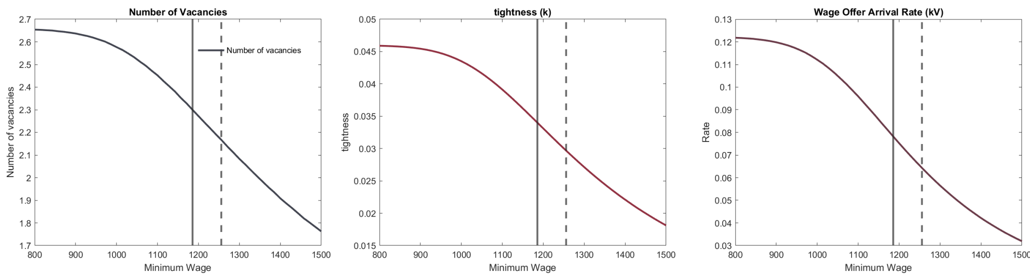
In the Metal and Non-metal industries, both unions and employers' associations would be willing to set the new level of minimum wages to bear around 12% of unemployment rate. The opposite results of the negotiations, the former lowering the wage floor and the latter increasing it, are due to differences in the lower support of wage distributions. The metal industry higher wage levels than the non-metal industry, as a result unions and employers in the metal industry prefer a moderate decrease of the minimum wage to improve employability, whereas inthe non-metal industry negotiators are willing to rise the level of the minimum wage because the start with lower wages and the effects on unemployment are milder.

The metal trading sector shows similar characteristics as the non-metal industry. However, in the metal trading sector the concentration of firm size is not as acute, which results in a lower cut-off threshold of firm size. Thus, workers and employers in smaller firms, compared to Non-metal industry, have a greater say in the negotiating table, workers are willing to rise the level of minimum wages despite the sharp increase in unemployment, because relative to the Non-metal industry, wages of workers in smaller firms experience an abrupt increase. In the Construction sector negotiators are willing to have a moderate decrease of the minimum wage, in this case even though insiders would like to decrease it even further, the fact that the cut-off threshold is already very low prevents actors from further cuts.

7.2 Competition and Employability

When imposing a wage floor there are two effects that impact in employability. The first one is mechanical, from the supply side, as the wage floor is increased the set of possible firms that are available for pairing is reduced, as shown in the theoretical part, only firms in the range $y \in [\underline{\hat{y}}(x), \overline{y}]$ will be available for workers with ability x, and the same mechanics are present in the other side of the market, where relatively low productive firms are not able to match workers with low ability to produce enough to pay for the minimum wage.

Figure 4: Fit of data distributions: wage cushion and firms



(a) Vacancy rate $\frac{v}{L}$ (b) Tightness: k (c) Offer arrival rate: kV

The second factor takes into account general equilibrium effects inside the labour market, the discussion goes along the same lines as with the unemployment rate. Figure 4 shows three graphs for the Metal Industry Sector: the number of vacancies, the tightness, and the wage offer arrival rate. In the first place, the number of vacancies is a decreasing function of the minimum wage. This is surprising, the introduction, or rise, of the minimum wage should have triggered a wave of laid-offs of relatively low-skill workers in small firms, which could have used the new jobs left vacant to search for high-skilled workers. The general equilibrium effect in the labour market explains this behaviour, the rise in wages and laid-offs due to the minimum wage, decreases dramatically the expected profits of holding a filled job, and firms close jobs at higher pace that filled jobs go vacant.

The tightness k is also a decreasing function of the minimum wage, the growing number of unemployed and vacancies outweigh the number of matches in the economy, as a result there are less matches with respect to the number of vacancies and searchers. Finally, to find out about the wage offer arrival rate we have to couple the two effects, clearly, the function will be decreasing as the other two are. As we see in the picture, social agents are willing to lower the level of the wage floor form €$1,256$ to €$1,190$, because the job arrival rate increases from 6.5% to 8%, and so the chances of being employed and further wage increases.

For the rest of the sectors the three graphs tells a similar story. The important graph is the wages offer arrival rate, which is left in figure 5 for the rest of the sectors. Basically, they tell the story along the same lines as the unemployed. Non-metal Industry and Metal Trading experience sharp decreases in the arrival rate from 10.7% to 7.5% and from 7% to 5% respectively. In contras the Metal Industry and Construction sectors experience moderate increases from 6.5% to 8% and from 6% to 6.5%.

(a) Metal Industry and Non-metal Industry

(b) Metal trading and Construction

Figure 5: Wage offer arrival rate as a function of the minimum wage for the four sectors. The solid line is the current minimum wage and the dashed line is the minimum wage under the new policy

7.3 Wage Inequality

The level of the minimum wage interferes with the wage distribution, and thus inequality, through three channels. The first one is mechanical, rising the minimum wage elevates the lower support of the wage distribution, then some workers will experience a pay rise and others will be dismissed, for a thorough impact of this mechanism on inequality see Dinardo, Fortin, and Lemieux (1996).

Another channel is the spillover effect. When minimum wages are increased, workers experience higher values as employed just because they now have the opportunity to work in high productivity firms earning no less than the minimum wage, and thus in the future they expect higher wages on average. In the end, workers demand a pay rise despite the minimum wage not being binding.

The last mechanism is related to the general equilibrium effect. Firms close vacancies which reduces the probability of wage offers arrivals. Therefore, UE and EE transitions, and wage increases will be reduced accordingly. Employed workers encounter less wage offers that end up in a wage increase, also they receive less wage offers from high productive firms. Thus, careers begin with a higher starting wage but at a cost of being stagnated for longer. Unemployed workers gain higher entry wages but at the expense of being less employable.

In figure 6 the effects of previous mechanisms are at play for two levels of the wage floor. As we can see, the wage distribution has a large mass point in the lower support and wage are concentrated right above this threshold for the reason explained before: Higher lower support, spillovers and higher entry wages. As we move along the distribution to the right tail, wages are being less concentrated because workers do not have as many opportunities. If the minimum wage were lowered all of this channels would be at work in the reverse order. Lower minimum means the lower support is going to be inferior, also poaching firms will be able to drag wages down further, which results in spillovers being attenuated. Higher rate of wage offer arrivals turns out in reduced entry wages. On the other end of the distribution, higher rates of wage offers end in more job-to-job transitions and pay rises.

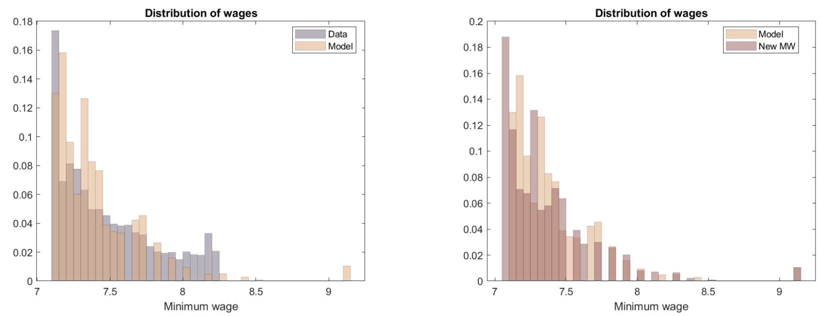
8 Discussion and Conclusions

This paper explores a direct, explicit relationship between collective agreements, minimum wages, and competition. Minimum wages set in collective agreements are used by participants in negotiations, unions and employer's associations in large firms, to drive small firms out of the market, reducing the level competition. Then, I build a search and matching model with on-the-job search and minimum wages in which unions and employers associations bargain for the optimal minimum wage level that fit them best.

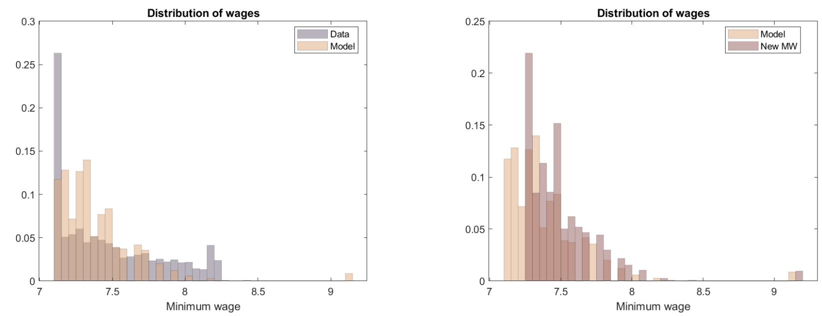
Surprisingly, I find that employers in large firms are interested in rising the level of minimum wage because they avoid competing with small firms for workers services, and as a result they do not have to grant pay rises as often. Another important result is that a policy maker could intervene to improve the situation, promoting participation in the negotiations so that the preferences of the whole labour force is taken into account.

Nonetheless, there are two features that should be taken into account for future research. First, several wage floors should be considered, so as to capture that different categories are subject to higher wage cushions. Another feature to be addressed is the large market power of firms, which remains unchallenged by the present work, however this problem could be tackled introducing multiworker firms and cost specific functions.

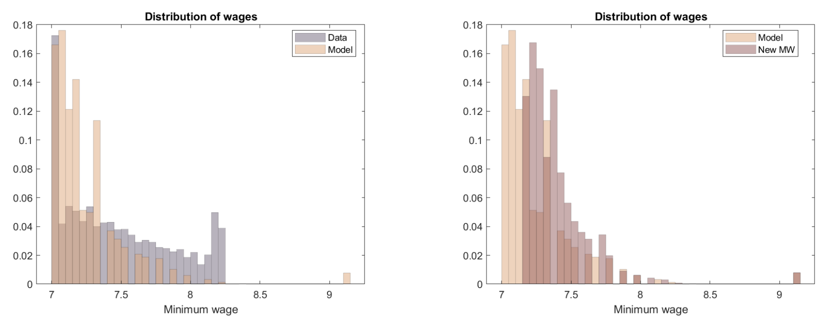
The main advantage of this work is that it can be readily be used in other environments where the access to administrative data and collective agreements is easily accessible.



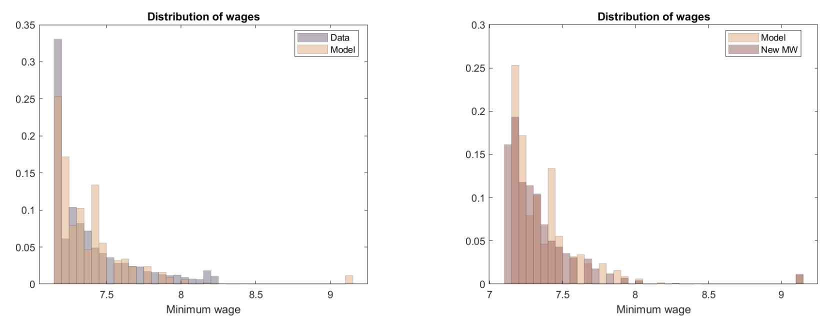
(a) Metal industry



(b) Non-metal industry



(c) Metal trading



(d) Construction

Figure 6: Wage distributions for the four sectors. Left panels: compare actual data with model. Right panels: compare model under the current level of minimum wages and the level under the new policy

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A Value Functions under the Minimum Wage

A.1 Unemployed

In this appendix expressions for equilibrium wages are determined for the basic model. I will closely follow the work of PV-R in deriving these analytical forms. First I will start setting the Value function of an unemployed worker in discrete time.

$$W_0(x) = bx\Delta + e^{-r\Delta}\left\{e^{-kV\Delta}W_0(x) + \left(1 - e^{-kV\Delta}\right)\int max\left[W_1\left(\phi_0(x,y),x,y\right), W_1\left(m,x,y'\right)\right]\frac{v(y')}{V}dy'\right\}$$

Since $W_1(w,x,y)$ is monotonically increasing in y there will be a threshold in which $W_1(m,x,y') \geq W_1(\phi_1(x,y),x,y)\ \ \forall y' \geq t_0$ or as implied by the lack of bargaining power of the worker $W_S(m,x,y') \geq 0$, hence the the threshold is implicitly defined as $W_S(m,x,t_0(x,y)) = 0$

$$W_0(x) = bx\Delta + e^{-r\Delta}\left\{e^{-kV\Delta}W_0(x) + \left(1 - e^{-kV\Delta}\right)\left(\int_{\underline{y}}^{t_0(x,y)} W_1\left(\phi_0(x,y),x,y\right)\frac{v(y')}{V}dy' + \int_{t_0(x,y)}^{\overline{y}} W_1\left(m,x,y'\right)\frac{v(y')}{V}dy'\right)\right\}$$

Because the lack of bargaining power of the worker $W_1(\phi_0(x,y),x,y) = W_0(x)$ we can rewrite

$$W_0(x) = bx\Delta + e^{-r\Delta}\left\{e^{-kV\Delta}W_0(x) + \left(1 - e^{-kV\Delta}\right)\left(\int_{\underline{y}}^{t_0(x,y)} W_0(x)\frac{v(y')}{V}dy' + \int_{t_0(x,y)}^{\overline{y}} W_1\left(m,x,y'\right)\frac{v(y')}{V}dy'\right)\right\}$$

rearranging

$$\left(1-e^{-(r+kV)\Delta}\right)W_0(x)=bx\Delta+e^{-r\Delta}\left(1-e^{-kV\Delta}\right)\left(\int_{\underline{y}}^{t_0(x,y)}W_0(x)\frac{v(y')}{V}dy'\right.$$
$$\left.+\int_{t_0(x,y)}^{\overline{y}}W_1\left(m,x,y'\right)\frac{v(y')}{V}dy'\right)$$

Dividing into Δ and taking the limit as Δ tends to zero, i.e. time is continuous, the expressions have the indeterminate form $\frac{0}{0}$. Then applying L'Hospital Rule

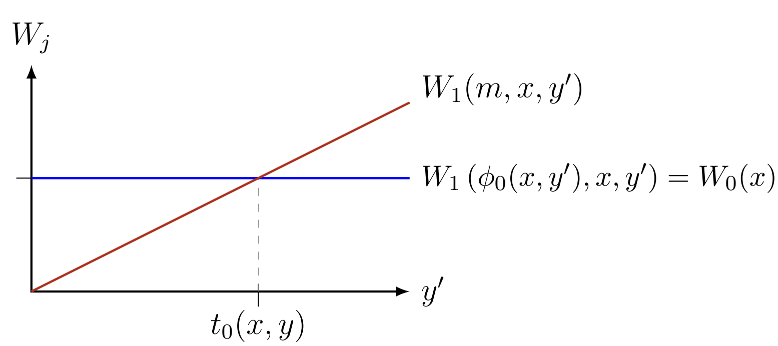
$$\left(r+kV\right)W_0(x)=bx+k\left(\int_{\underline{y}}^{t_0(x,y)}W_0(x)v(y')dy'\right.$$
$$\left.+\int_{t_0(x,y)}^{\overline{y}}W_1\left(m,x,y'\right)v(y')dy'\right)$$

rearranging

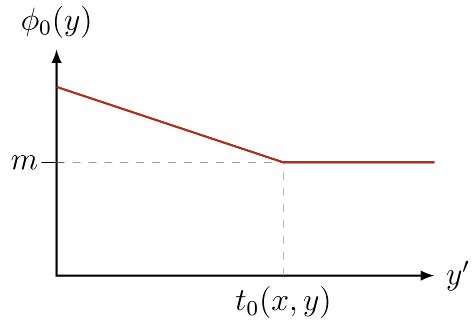
$$\left(r+k\overline{V}\left(t_0(x,y)\right)\right)W_0(x)=bx+k\int_{t_0(x,y)}^{\overline{y}}W_1\left(m,x,y'\right)v(y')dy'$$
$$rW_0(x)=bx+k\int_{t_0(x,y)}^{\overline{y}}W_S\left(m,x,y'\right)v(y')dy'$$

For computational purposes it is more convenient to write the equation as

$$rW_0(x)=bx+k\int max\left[W_S\left(m,x,y'\right),0\right]v(y')dy'$$



Threshold $t_0(x,y)$



Wage offer $\phi_0(x, y)$

A.2 Employed

Turning to the employed workers of any type, the value function of an x employee working on a firm of type y and earning the wage $w \leq xy$ is derived in the same fashion. Because there is a minimum wage in place, for a particular x, the lower bound of firms might change being $\underline{y}^* = min[\underline{y}, \hat{y}(x)]$, where $\hat{y}(x)$ is minimum viable productivity of a firm to hire a worker of type x when the minimum wage is binding.

$$
\begin{aligned}
&W_1(w, x, y) \\
&\quad = w\Delta + e^{-r\Delta}\Big\{\Big(1 - e^{-\delta\Delta}\Big)W_0(x) + e^{-\delta\Delta}\Big[e^{-skV\Delta}W_1(w, x, y) \\
&\quad + \Big(1 - e^{-skV\Delta}\Big)\Bigg(\int_{\underline{y}^*}^{q(w,x,y)} W_1(w, x, y)\frac{v(y')}{V}dy' + \int_{q(w,x,y)}^{y} W_1(xy', x, y)\frac{v(y')}{V}dy' \\
&\quad + + \int_{y}^{t_1(x,y)} W_1(xy, x, y')\frac{v(y')}{V}dy' + \int_{t_1(x,y)}^{\overline{y}} W_1(m, x, y')\frac{v(y')}{V}dy'\Bigg)\Big]\Big\}
\end{aligned}
$$

Rearranging

$$
\begin{aligned}
&\Big(1 - e^{-(r+\delta+skV)\Delta}\Big)W_1(w, x, y) \\
&\quad = w\Delta + e^{-r\Delta}\Big\{\Big(1 - e^{-\delta\Delta}\Big)W_0(x) \\
&\quad + \Big(1 - e^{-skV\Delta}\Big)\Bigg(\int_{\underline{y}^*}^{q(w,x,y)} W_1(w, x, y)\frac{v(y')}{V}dy' + \int_{q(w,x,y)}^{y} W_1(xy', x, y)\frac{v(y')}{V}dy' \\
&\quad + + \int_{y}^{t_1(x,y)} W_1(xy, x, y')\frac{v(y')}{V}dy' + \int_{t_1(x,y)}^{\overline{y}} W_1(m, x, y')\frac{v(y')}{V}dy'\Bigg)\Big\}
\end{aligned}
$$

Dividing into Δ and taking the limit as Δ tends to zero, i.e. time is continuous, the expressions have the indeterminate form $\frac{0}{0}$. Then applying L'Hospital Rule

$$
\begin{aligned}
(r+\delta+skV)\, W_1\,(w,x,y) \\
&= w + \delta W_0(x) \\
&+ sk \int_{\underline{y}^*}^{q(w,x,y)} W_1(w,x,y) v(y')dy' \\
&+ sk \int_{q(w,x,y)}^{y} W_1(xy',x,y) v(y')dy' \\
&+ sk \int_{y}^{t_1(x,y)} W_1(xy,x,y') v(y')dy' \\
&+ sk \int_{t_1(x,y)}^{\overline{y}} W_1(m,x,y') v(y')dy'
\end{aligned}
$$

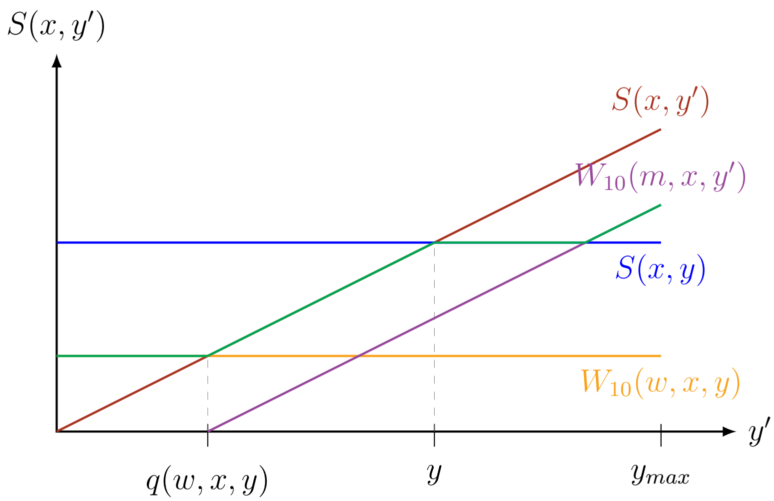
Now, because $\Pi_0(y) = 0,\ \forall y$ then $W_1(xy,x,y) = P(x,y)$, substituting this into the previous equation

$$
\begin{aligned}
(r+\delta+skV)\, W_1\,(w,x,y) \\
&= w + \delta W_0(x) \\
&+ sk \int_{\underline{y}^*}^{q(w,x,y)} W_1(w,x,y) v(y')dy' \\
&+ sk \int_{q(w,x,y)}^{y} P(x,y') v(y')dy' \\
&+ sk \int_{y}^{t_1(x,y)} P(x,y) v(y')dy' \\
&+ sk \int_{t_1(x,y)}^{\overline{y}} W_1(m,x,y') v(y')dy'
\end{aligned}
$$

Subtracting $(r + \delta + skV)\, W_0(x)$ from both sites and rearranging

$$\begin{aligned}\left(r + \delta + sk\overline{V}(q)\right) W_S\,(w, x, y) & \\ &= w - rW_0(x) \\ &+ sk \int_{q(w,x,y)}^{y} S(x, y')v(y')dy' \\ &+ sk \int_{y}^{t(x,y)} S(x, y)v(y')dy' \\ &+ sk \int_{t_1(x,y)}^{\overline{y}} W_S(m, x, y')v(y')dy'\end{aligned}$$

Following the picture we can rewrite the expression for computational purposes and thus



Threshold $t_1(x, y)$

avoid calculating $q(w, x, y)$ for every wage

$$\begin{aligned}&(r\ +\delta + skV)\, W_1\,(w, x, y) \\ &\quad = w + \delta W_0(x) \\ &\quad + sk \int max\left\{min\left[max\left(W_S(w, x, y), S(x, y')\right), S(x, y)\right], W_S(m, x, y')\right\} v(y')dy'\end{aligned}$$

And the threshold $t_1(x, y)$ is defined as

$$W_1(xy, x, y) = W_1(m, x, t_1(x, y)) \;\; or$$
$$S(x, y) = W_S(m, x, t_1(x, y))$$

A.3 Value of a Match and Surplus

Define the value of a Match and Surplus as $P(x, y) = \Pi_1(w, x, y) + W_1(w, x, y)$ and $S(x, y) = P(x, y) - W_0(x)$ respectively. Because there is a minimum wage in place, for a particular x, the lower bound of firms might change being $\underline{y}^* = min[\underline{y}, \hat{y}(x)]$, where $\hat{y}(x)$ is minimum viable productivity of a firm to hire a worker of type x when the minimum wage is unemployment binding. again we start setting the value of a match as

$$\begin{aligned}
&P(x, y) \\
&\quad = yx\Delta + e^{-r\Delta}\left\{\left(1 - e^{-\delta\Delta}\right)W_0(x)\right. \\
&\quad + e^{-\delta\Delta}\left[e^{-skV\Delta}P(x, y) + \left(1 - e^{-skV\Delta}\right)\left(\int_{\underline{y}^*}^{q} P(x, y)\frac{v(y')}{V}dy'\right.\right. \\
&\quad + \left.\left.\left.\int_{q}^{y} P(x, y)\frac{v(y')}{V}dy' + \int_{y}^{t_1(x,y)} P(x, y)\frac{v(y')}{V}dy' + \int_{t_1(x,y)}^{\overline{y}} W_1(m, x, y')\frac{v(y')}{V}dy'\right)\right]\right\}
\end{aligned}$$

rearranging

$$\begin{aligned}
&\left(1 - e^{-(r+\delta+skV)\Delta}\right)P(x, y) \\
&\quad = yx\Delta + e^{-r\Delta}\left\{\left(1 - e^{-\delta\Delta}\right)W_0(x)\right. \\
&\quad + \left.e^{-\delta\Delta}\left[\left(1 - e^{-skV\Delta}\right)\left(\int_{\underline{y}^*}^{t_1(x,y)} P(x, y)\frac{v(y')}{V}dy' + \int_{t_1(x,y)}^{\overline{y}} W_1(m, x, y')\frac{v(y')}{V}dy'\right)\right]\right\}
\end{aligned}$$

Dividing into Δ and taking the limit as Δ tends to zero, i.e. time is continuous, the expressions have the indeterminate form $\frac{0}{0}$. Then applying L'Hospital Rule

$$\begin{aligned}(r+\delta+skV)\,P(x,y)& \\ &= yx+\delta W_0(x) \\ &+ sk\int_{\underline{y}^*}^{t_1(x,y)} P(x,y)v(y')dy' + sk\int_{t_1(x,y)}^{\overline{y}} W_1(m,x,y')v(y')dy'\end{aligned}$$

Rearranging

$$\begin{aligned}\left(r+\delta+sk\overline{V}(t_1)\right)P(x,y)& \\ &= yx+\delta W_0(x) \\ &+ sk\int_{t_1(x,y)}^{\overline{y}} W_1(m,x,y')v(y')dy'\end{aligned}$$

We just need to subtract $\left(r+\delta+sk\overline{V}(t_1)\right)W_0(x)$ to both sites to have a close expression for the surplus

$$\left(r+\delta+sk\overline{V}(t_1)\right)S(x,y) = yx - rW_0(x) + sk\int_{t_1(x,y)}^{\overline{y}} W_S(m,x,y')v(y')dy'$$

Rewriting for computational purposes

$$(r+\delta+skV)\,S(x,y) = yx - rW_0(x) + sk\int max\left[S(x,y), W_S(m,x,y')\right]v(y')dy'$$

Remember that I am assuming that $\Pi_0(y)=0,\ \forall y$

B Equilibrium Wage Determination under The Minimum wage

B.1 Employed

Turning to the employed workers of any type, the value function of an x employee working on a firm of type y and earning the wage $w \leq xy$ is set as to equal the wage, plus the value as unemployed with a laid-off rate of δ and offers from outside firms, accruing at a rate skV. When a worker is poached by a firm with productivity $\tilde{y} \leq q$ then $\phi(x, \tilde{y}, y) \leq w$, the poacher does not even reach the necessary productivity to hire the worker and make profits. At this point the minimum viable productivity that the firm has to have in order to provoke a wage increase is defined implicitly in the usual way as $\phi\left(x, q(x, w, y), y\right) = w$. If the offering firm has productivity $[q, y]$, the worker will receive a wage increase due to the Bertrand competition. In the case where the firm would have productivity higher than y, up to a threshold $t_1(x, y)$, the worker will switch jobs, the commonly known interplay between the wage offer and future wages increases plays its role, leaving the discounted future value of wealth fixed in this interval, i.e. the value function reaches a plateau since the firm is able to extract all the value from the match, thus higher productivity (and more likely future wage increases) is offset by a reduction in the wage offered. Now, offers from firms with higher productivity than $t_1(x, y)$ cannot reduce the offer made to the worker to extract all the match value, since the minimum wage acts as a lower bound for wages, in other words the minimum wage is binding. From the onset I will assume that this threshold exists and is unique as will be shown later. The value function of the worker is left as

$$
\begin{aligned}
\left[r+\delta+sk\overline{V}\left(q(w,x,y)\right)\right]W_1(w,x,y) &= w+\delta W_0(x) \\
&+ sk\int_{q(w,x,y)}^{y} W_1(xy',x,y')\; v(y')dy' \\
&+ sk\int_{y}^{t_1(x,y)} W_1(xy,x,y)\; v(y')dy' \\
&+ sk\int_{t_1(x,y)}^{\overline{y}} W_1(m,x,y')\; v(y')dy' \qquad (5)
\end{aligned}
$$

Where the threshold is defined as $W_1(xy,x,y)=W_1\left(m,x,t_1(x,y)\right)$

Adding and subtracting $W_1(xy,x,y)$ in the range $[t_1(x,y),\overline{y}]$ and rearranging terms we have

$$
\begin{aligned}
\left[r+\delta+sk\overline{V}\left(q(w,x,y)\right)\right]W_1(w,x,y) &= w+\delta W_0(x) \\
&+ s\kappa\int_{q(w,x,y)}^{y} W_1(xy',x,y')\; v(y')dy' \\
&+ sk\int_{y}^{\overline{y}} W_1(xy,x,y)\; v(y')dy' \\
&+ sk\int_{t_1(x,y)}^{\overline{y}} W_1(m,x,y')-W_1(xy,x,y)\; v(y')dy'
\end{aligned}
$$

The last term in the equation is the increase in utility granted by the minimum wage. Now, imposing $w=xy$ implies that $q(xy,x,y)=y$, introducing these concerns into the previous equation:

$$
\begin{aligned}
\left[r+\delta+sk\overline{V}\left(y\right))\right]W_1(xy,x,y) &= xy+\delta W_0(x) \\
&+ \underbrace{sk\int_{y}^{\overline{y}} W_1(xy,x,y)v(y')dy'}_{sk\overline{V}(y))W_1(xy,x,y)} \\
&+ sk\int_{t_1(x,y)}^{\overline{y}} W_1(m,x,y') - \underbrace{W_1(xy,x,y)}_{W_1(xy,x,y)\,\overline{V}(t_1(x,y))}\quad v(y')dy'
\end{aligned}
$$

Which results in

$$\left[r + \delta + sk\overline{V}\left(t_1(x,y)\right)\right] W_1(xy,x,y) = xy + \delta W_0(x) + sk \int_{t_1(x,y)}^{\overline{y}} W_1(m,x,y')\ v(y')dy'$$

This last expression can be derived with respect to y to have a specific expression of the derivative, then differentiating implicitly

$$\begin{aligned}-skv\left(t_1(x,y)\right)t_1'(x,y)W_1(xy,x,y) + \left[r + \delta + sk\overline{V}\left(t_1(x,y)\right)\right] W_1'(xy,x,y) \\ = x - sk\underbrace{W_1(m,x,t_1(x,y))}_{W_1(xy,x,y)}v\left(t_1(x,y)\right)t_1'(x,y)\end{aligned}$$

Solve for $W_1'(xy,x,y)$

$$W_1'(xy,x,y) = \frac{x}{\left[r + \delta + sk\overline{V}\left(t_1(x,y)\right)\right]} \tag{6}$$

Integrating (5) by parts, we have

$$\begin{aligned}\Big[r + \delta + &sk\overline{V}\left(q(w,x,y)\right)\Big] W_1(w,x,y) = w + \delta W_0(x) \\ &+ skW_1\left(xy,x,y\right)V(y) - skW_1\left(w,x,y\right)V\left(q\right) - sk\int_q^y W_1'\left(xy',x,y'\right)V(y')\ dy' \\ &+ skW_1(xy,x,y)\left[V\left(t_1(x,y)\right) - V(y)\right] \\ &+ skW_1\left(m,x,\overline{y}\right)V - sk\underbrace{W_1\left(m,x,t_1(x,y)\right)}_{W_1(xy,x,y)}V\left(t_1(x,y)\right) - \int_{t_1(x,y)}^{\overline{y}} W_1'\left(m,x,y'\right)\ V(y')dy'\end{aligned}$$

Which after cancelling terms and rearranging, results in

$$\begin{aligned}[r+\delta]W_1(w,x,y) = w + \delta W_0(x) \\ + sk\left[W_1(xy,x,y)V - W_1(w,x,y)V - \int_q^y W_1'(xy',x,y')V(y')\ dy'\right] \\ + sk\left[W_1(m,x,\overline{y})V - \underbrace{W_1(xy,x,y)V}_{W_1(m,x,t_1(x,y))} - \int_{t_1(x,y)}^{\overline{y}} W_1'(m,x,y')V(y')\ dy'\right]\end{aligned}$$

Now, thanks to the fundamental theorem of calculus we get to the usual expression

$$[r+\delta]W_1(w,x,y) = w + \delta W_0(x) + sk\int_q^y W_1'(xy',x,y')\overline{V}(y')\,dy' + sk\int_{t_1(x,y)}^{\overline{y}} W_1'(m,x,y')\overline{V}(y')\,dy' \tag{7}$$

However this expression is not very intuitive, instead it would be better to have the value function defined in the whole support of firms, for which the following change of variables can be performed

$$\left.\begin{aligned} y' &= t_1(x,z) \\ dy' &= t_1'(x,z)dz \end{aligned}\right\} W_1(m,x,y') = W_1(xz,x,z) \Rightarrow W_1(m,x,t_1(x,z)) = W_1(xz,x,z)$$

Deriving with respect to z

$$W_1'\left(m,x,t_1(x,z)\right)t_1'(x,z)dz = W_1'(xz,x,z)$$

Making use of (6), (7) and the previous expression the final form follows

$$[r+\delta]W_1(w,x,y) = w + \delta W_0(x) + skx\int_q^y \frac{\overline{V}(y')}{\left[r+\delta+sk\overline{V}\left(t_1(x,y)\right)\right]}\,dy' + skx\int_y^{\overline{y}} \frac{\overline{V}\left(t_1(x,y')\right)}{\left[r+\delta+sk\overline{V}\left(t_1(x,y)\right)\right]}\,dy' \tag{8}$$

With this general expression at hand we are ready to compute a particular expression of the wages. Start with the fact that an outside offer coming from a firm $\tilde{y} < y$ should comply with the equality

$$W_1\left(\phi(x,\tilde{y},y),x,y\right) = max\left[W_1\left(x\tilde{y},x,\tilde{y}\right), W_1\left(m,x,y\right)\right]$$

Notice that because $\tilde{y} < y$, m is not going to be binding and the maximum function will result in $W_1(x\tilde{y}, x, \tilde{y})$. Then, taking the specific forms of the value functions derived in (8) at the particular wages, we can write

$$\phi(x,\tilde{y},y) + \delta W_0(x) + skx\left\{\int_{\tilde{y}}^{y} \frac{\overline{V}(y')dy'}{\left[r+\delta+sk\overline{V}(t_1(x,y))\right]} + \int_{y}^{\overline{y}} \frac{\overline{V}(t_1(x,y')dy')}{\left[r+\delta+sk\overline{V}(t_1(x,y))\right]}\right\}$$
$$= x\tilde{y} + \delta W_0(x) + skx\left\{\int_{\tilde{y}}^{\tilde{y}} \frac{\overline{V}(y')dy'}{\left[r+\delta+sk\overline{V}(t_1(x,y))\right]} + \int_{\tilde{y}}^{\overline{y}} \frac{\overline{V}(t_1(x,y')dy')}{\left[r+\delta+sk\overline{V}(t_1(x,y))\right]}\right\}$$

Which after rearranging becomes

$$\underbrace{\phi(x,\tilde{y},y)}_{\text{wage offer}} = \underbrace{x\tilde{y}}_{\text{max. productivity of the match}} - \underbrace{skx\int_{\tilde{y}}^{y} \frac{\overline{V}(y')dy'}{\left[r+\delta+sk\overline{V}(t_1(x,y))\right]}}_{\text{Trade off of lower wages for future increases}} + \underbrace{skx\int_{\tilde{y}}^{y} \frac{\overline{V}(t_1(x,y'))\,dy'}{\left[r+\delta+sk\overline{V}(t_1(x,y))\right]}}_{\text{Extra rent granted by imposing a min. wage}}$$
$$= x\left\{\tilde{y} + sk\int_{\tilde{y}}^{y} \frac{\overline{V}(t_1(x,y')) - \overline{V}(y')}{\left[r+\delta+sk\overline{V}(t_1(x,y))\right]}dy'\right\}$$

B.2 Unemployed

As in the case without minimum wages we just need to define the minimum viable productivity of a firm $\hat{y}(x)$ as the value that leaves the worker indifferent between looking for a job and working; and at the same time makes the surplus of the match equal to zero, if the surplus of the match were not zero at the firm with the lowest viable productivity, any other firm with marginally lower productivity could make an offer to the worker and make profits at the same time. Since, there is no restriction on how low the marginal productivity of a firm can be, this is a contradiction, and thus:

$$P(x,\hat{y}(x)) = W_1(m,x,\hat{y}(x)) = W_0(x;m)$$

Where $\hat{y}(x) = \frac{m}{x}$

Proof:

$$
\begin{aligned}
P(x, \hat{y}(x)) &= x\hat{y}(x) + sk\int_{\hat{y}(x)}^{t_1} P(x,y)v(y')dy' + sk\int_{t_1}^{\overline{y}} W_1(m,x,y')v(y')dy' \\
W_1(m, x, \hat{y}(x)) &= m + sk\int_{\hat{y}(x)}^{t_1} P(x,y)v(y')dy' + sk\int_{t_1}^{\overline{y}} W_1(m,x,y')v(y')dy'
\end{aligned}
$$

Equating terms we arrive at $x\hat{y}(x) = m$, hence the result $\hat{y}(x) = \frac{m}{x}$.

And the entry wage will be m in any case, $\phi_0(x, \tilde{y}) = m$.

B.3 High-skill workers

High-skill workers will show similar expressions as those worked out of low-skill ones. For employed workers the expression for salaries will remain unchanged, since the process for poaching workers is basically the same. Also, since the effective minimum wage $\hat{y}(x)$ is below the minimum viable productivity of a firm, $\hat{y}(x) \leq \underline{y}$, the support of the distribution will remain unchanged. What is likely to change is the support of the distribution of wages since now the entry wage will be the largest between $\phi_0(x, y; m) = [\phi_0(x, y), m]$, depending on the productivity of the initial poacher. Then the expression for entry wages will be a piece-wise function of the form

$$
\phi_0(\epsilon, y; m) = \left\{ \begin{matrix} x\left\{b + sk\int_{\underline{y}}^{y} \frac{\overline{V}(t_1(x,y')) - \overline{V}(y')}{\left[\rho+\delta+sk\overline{V}(t_1(x,y))\right]}dy'\right\} & \text{if} \quad y' < t_1(x,y)) \\ m & \text{if} \quad y' \geq t_1(x,y)) \end{matrix} \right\}
$$

C Mathematical Proofs

C.1 Lemma 1

Making use of LEMMA.2 we can equate $P\left(x, \underline{\hat{y}}(x)\right) = W_1\left(\phi_0\left(x, \underline{\hat{y}}(x)\right), x, \underline{\hat{y}}(x)\right)$, which means that

$$\underline{\hat{y}}(x)x + \delta W_0(x) + sk\int_{\underline{\hat{y}}(x)}^{t(x,y)} P(x,y)v(y')dy' + sk\int_{t(x,y)}^{\overline{y}} W_1(m,x,y')v(y')dy'$$
$$= \phi_0\left(x, \underline{\hat{y}}(x)\right) + \delta W_0(x) + sk\int_{\underline{\hat{y}}(x)}^{t(x,y)} P(x,y)v(y')dy' + sk\int_{t(x,y)}^{\overline{y}} W_1(m,x,y')v(y')dy'.$$

And after cancelling terms we arrive at:

$$\underline{\hat{y}}(x)x = \phi_0\left(x, \underline{\hat{y}}(x)\right).$$

For convenience define the threshold x' such that $\phi_0\left(x', \underline{y}\right) = m$. There are two cases of interest:

CASE.1: $x < x'$

$$\phi_0\left(x', \underline{\hat{y}}(x)\right) = m \Leftrightarrow \underline{\hat{y}}(x) = \frac{m}{x}.$$

CASE.2: $x \geq x'$

$$\phi_0\left(x', \underline{y}\right) > m \Leftrightarrow \underline{\hat{y}}(x) = \underline{y}.$$

C.2 Lemma 2

Free-entry implies that the firm with the minimum viable productivity to hire a worker cannot make any surplus out of the match, i.e. $\Pi_1\left(x, \underline{\hat{y}}(x)\right) = 0$, otherwise a firm with marginally less productivity could enter the market, hire a worker and make profits, being a contradiction; then $P\left(x, \underline{\hat{y}}(x)\right) = W_1\left(\phi_0\left(x, \underline{\hat{y}}(x)\right), x, \underline{\hat{y}}(x)\right)$.

With respect to $W_1\left(\phi_0\left(x, \underline{\hat{y}}(x)\right), x, \underline{\hat{y}}(x)\right) = W_0(x,m)$, the same argument along the above lines can be devised. If $W_1\left(\phi_0\left(x, \underline{\hat{y}}(x)\right), x, \underline{\hat{y}}(x)\right) = P\left(x, \underline{\hat{y}}(x)\right) > W_0(x,m)$ then another firm with marginally less productivity could enter and make profits, once again a contradiction.

D Steady State Distributions

D.1 Base Model Distributions

In this appendix detailed derivations for $v(y)$, $u(x)$ and $h(x,y)$ are worked out. We start with the balance conditions, which are no more that accounting identities, that are met in every point in time

$$\int h_t(x,y')dy' + u_t(x) = l_t(x)$$
$$\int h_t(x',y)dx' + v_t(y) = n_t(y)$$

and flow equations in discrete time.

$$\begin{aligned}
h_{t+1}(x,y) &= h_t(x,y) \quad +kv_t(y)u_t(x)\Delta + sk\int_{\underline{y}}^{y} h_t(x,y')dy'v_t(y)\Delta \quad -\delta h_t(x,y)\Delta - sk\int_{y}^{\overline{y}} v_t(y')dy'h_t(x,y)\Delta \\
u_{t+1}(x) &= u_t(x) \qquad -kV_t u_t(x)\Delta \qquad\qquad +\delta h_{x,t}(x)\Delta \\
v_{t+1}(y) &= v_t(y) \qquad -kv(y)U_t\Delta - sk\int_{\underline{y}}^{y} h_{y,t}(y')dy'v_t(y)\Delta \quad +\delta h_{y,t}(y)\Delta + sk\int_{y}^{\overline{y}} v_t(y')dy'h_{y,t}(y)\Delta
\end{aligned}$$

Where $h_{x,t}(x) = \int_{\underline{y}}^{\overline{y}} h_t(x,y')dy'$ and $h_{y,t}(y) = \int_{\underline{x}}^{\overline{x}} h_t(x',y)dx'$. The expressions are easier to work with in continuous time so I rearrange stocks to the LHS and flows to the RHS, Divide by Δ and take the limit as $\Delta \to 0$ to have

$$\begin{aligned}
\dot{h}_t(x,y) &= kv_t(y)u_t(x) \quad +sk\int_{\underline{y}}^{y} h_t(x,y')dy'v_t(y) - \delta h_t(x,y) \quad -sk\int_{y}^{\overline{y}} v_t(y')dy'h_t(x,y) \\
\dot{u}_t(x) &= -kV_t u_t(x) \qquad + \delta h_{x,t}(x) \\
\dot{v}_t(y) &= -kv_t(y)U_t \quad -sk\int_{\underline{y}}^{y} h_{y,t}(y')dy'v_t(y) + \delta h_{y,t}(y) \quad +sk\int_{y}^{\overline{y}} v_t(y')dy'h_{y,t}(y)
\end{aligned}$$

Before working out specific expressions for every type of firm and worker, it will be useful to calculate aggregate balance conditions, just aggregate over the set of firm productivities and worker abilities to have

$$H_t + U_t = L_t$$

$$H_t + V_t = N_t$$

Where $H_t = \int\int h_t(x', y')dx'dy'$. L_t is exogenous and given N_t, V_t is pinned down, at this point both conditions are knotted by H_t, so we can write

$$L_t - U_t = N_t - V_t$$

And deriving with respect to time we have that $\dot{U}_t = \dot{V}_t$. Also, we need to have the expressions for the aggregate flows. Integrate $v(y)$, $u(x)$ and $h(x, y)$ over the variables that they depend on. Furthermore, in the aggregate all the workers that quit are the same as those who are poached, i.e. $\int_{\underline{y}}^{y} h_t(x, y')dy'v_t(y) = \int_{y}^{\overline{y}} v_t(y')dy'h_t(x, y)$, hence

$$\left.\begin{aligned} \dot{H}_t &= kV_tU_t - \delta H_t \\ \dot{U}_t &= -kV_tU_t + \delta H_t \\ \dot{V}_t &= -kV_tU_t + \delta H_t \end{aligned}\right\} \overset{SS}{\Longrightarrow} \delta H_t = kV_tU_t$$

Since we are in the S.S., time dependence can be dropped from the notation. Now, Plug the aggregate balance conditions to have H as a function of N, k (endogenous objects) and δ, L (parameters), $\delta H = k(N - H)(L - H)$, this is a quadratic equation on H

$$kH^2 - (\delta + kL + kN)\, H + kLN = 0$$

Which solves as

$$H = \frac{1}{2}\left\{\left(\frac{\delta}{k} + L + N\right) - \sqrt{\left(\frac{\delta}{k} + L + N\right)^2 - 4LN}\right\}$$

Below, it is the proof of why only the negative part is taken. First consider the positive part, i.e.

$$H(N)=\frac{1}{2}\left\{\left(\frac{\delta}{k}+L+N\right)+\sqrt{\left(\frac{\delta}{k}+L+N\right)^{2}-4LN}\right\}>\frac{1}{2}\left\{\left(\frac{\delta}{k}+L+N\right)+\sqrt{\left(\frac{\delta}{k}+L+N\right)^{2}}\right\}\Leftrightarrow$$
$$\frac{1}{2}\left\{\left(\frac{\delta}{k}+L+N\right)+\sqrt{\left(\frac{\delta}{k}+L+N\right)^{2}-4LN}\right\}>\left(\frac{\delta}{k}+L+N\right)$$

Which means that the number of employed is higher than the number of people in the economy, an absurdity. Turning to the negative part, I would like to work out the maximum and minimum values as a function of N. The minimum value can be easily worked out as

$$H\left(0\right)=\frac{1}{2}\left\{\left(\frac{\delta}{k}+L\right)-\sqrt{\left(\frac{\delta}{k}+L\right)^{2}}\right\}=0$$

And the maximum

$$\lim_{N\to\infty}H(N)=\lim_{N\to\infty}\frac{1}{2}\left\{\left(\frac{\delta}{k}+L+N\right)-\sqrt{\left(\frac{\delta}{k}+L+N\right)^{2}-4LN}\right\}$$

Which is undetermined, dividing and multiplying by the complement and later on dividing by N in the numerator and denominator we have

$$\begin{aligned}\lim_{N\to\infty}H(N)&=\lim_{N\to\infty}\frac{1}{2}\left\{\frac{4LN}{\left(\frac{\delta}{k}+L+N\right)+\sqrt{\left(\frac{\delta}{k}+L+N\right)^{2}-4LN}}\right\}\\&=\lim_{N\to\infty}\frac{1}{2}\left\{\frac{4L}{\left(\frac{\delta}{Nk}+\frac{L}{N}+1\right)+\sqrt{\left(\frac{\delta}{Nk}+\frac{L}{N}+1\right)^{2}+\frac{4L}{N}}}\right\}\\&=\frac{1}{2}\left\{\frac{4L}{1+\sqrt{1}}\right\}=L\end{aligned}$$

Which means that as the number of firms tends to infinity the number of employed workers tends to the number of people in the economy. Also, it would be convenient to check if the function is increasing in the whole domain

$$\frac{\partial H}{\partial N} = \frac{1}{2}\left\{1 - \frac{2\left(\frac{\delta}{k} + L + N\right) - 4L}{\sqrt{\left(\frac{\delta}{k} + L + N\right)^2 - 4LN}}\right\} > 0$$

Using the balance conditions U and V can easily be derived. With this expressions at hand we can work out their desegregated counterparts. First, consider the S.S. and drop the time dependence,so

$$\begin{aligned}
0 &= kv(y)u(x) && +sk\int_{\underline{y}}^{y} h(x,y')dy'v(y) - \delta h(x,y) && -sk\int_{y}^{\overline{y}} v(y')dy'h(x,y) \\
0 &= -kVu(x) && \qquad\qquad + \delta h_x(x) && \\
0 &= -kv(y)U && -sk\int_{\underline{y}}^{y} h_y(y')dy'v(y) + \delta h_y(y) && +sk\int_{y}^{\overline{y}} v(y')dy'h_y(y)
\end{aligned}$$

Now, consider the last expression and rearrange such that U-to-H and H-to-U flows are on the LHS and H-to-H flows are in the RHS. Also, for notational convenience let's write $F_z(z) = \int_{\underline{z}}^{z} f_z(z')dz'$ for any function over the z-characteristic and denote its complement counterpart as $\overline{F}_z(z) = F_z(\overline{z}) - F_z(z) = \int_{z}^{\overline{z}} f_z(z')dz'$

$$kv(y)U - \delta h_y(y) = -skH_y(y)v(y) + sk\overline{V}(y)h_y(y)$$

Integrate both sites from $\underline{y}$ to y to have

$$kV(y)U - \delta H_y(y) = skH_y(y)\overline{V}(y) \tag{9}$$

And Integrate the balance condition from $\underline{y}$ to y to have

$$H_y(y) + V(y) = N(y)$$

Then solve last expression for V(y) and plug it into the aggregate flow equation for vacancies to arrive at

$$k\left(N(y)-H_y(y)\right)U-\delta H_y(y)=skH_y(y)\left(V-N(y)+H_y(y)\right)$$
$$kUN(y)-kUH_y(y)-\delta H_y(y)=skVH_y(y)-skN(y)H_y(y)+skH^2(y)$$
$$skH^2(y)+\left(\delta+kU+skV-skN(y)\right)H_y(y)-kUN(y)=0$$

Again, this is a quadratic equation in $H_y(y)$, which depends solely on k and $N(y)$ and the rest of variables have been previously worked out, as we can see below

$$\underbrace{sk}_{A}H_y^2(y)+\underbrace{(\delta+kU+skV-skN(y))}_{B(N(y))}H_y(y)-\underbrace{kUN(y)}_{C(N(y))}=0$$

Then

$$H_y(y)=\frac{-B\left(N(y)\right)+\sqrt{B^2\left(N(y)\right)+4AC\left(N(y)\right)}}{2A}$$

The negative part can safely be discarded as

$$H_y(y)=\frac{-B\left(N(y)\right)-\sqrt{B^2\left(N(y)\right)+4AC\left(N(y)\right)}}{2A}<\frac{-B\left(N(y)\right)-\sqrt{B^2\left(N(y)\right)}}{2A}=-\frac{2B\left(N(y)\right)}{2A}<0$$

Whereas the positive part is always greater than zero

$$H_y(y)=\frac{-B\left(N(y)\right)+\sqrt{B^2\left(N(y)\right)+4AC\left(N(y)\right)}}{2A}>\frac{-B\left(N(y)\right)+\sqrt{B^2\left(N(y)\right)}}{2A}=0$$

Now, derive the quadratic equation implicitly with respect to to y to find $h_y(y)$

$$2skH_y(y)h_y(y)+\left(\delta+kU+skV-skN(y)\right)h_y(y)-skn(y)H_y(y)-kUn(y)=0$$

Solving for $h_y(y)$

$$h_y(y) = \frac{kU + skH_y(y)}{(\delta + skV + skH_y(y) - skN(y)) + (kU + skH_y(y))} \cdot n(y)$$

Where

- $(\delta + skV + skH_y(y) - skN(y)) = \left(\delta + sk\overline{V}(y)\right)$: are flows out of $H_y(y)$ and
- $kU + skH_y(y)$: are flows into $H_y(y)$

It's worth noting that $H_y(y)$ depends on $N(y)$ and so does $h_y(y)$.

At this point we are ready to come with an expression for $v(y)$. Consider again the expression coming from the integrated flow of vacancies in the S.S.

$$kV(y)U - \delta H_y(y) = skH_y(y)\overline{V}(y)$$

It is just left to solve for $V(y)$ and derive to reach the desire result

$$V(y) = \frac{\delta + skV}{kU + skH_y(y)} H_y(y)$$

And deriving

$$v(y) = \frac{\delta + skV}{(kU + skH_y(y))^2} kUh_y(y)$$

Which is not very intuitive. In order to have an expression in terms of flows, change $h_y(y)$ by its last derived expression; plug the definition of $V(y)$ and use the integrated flow of vacancies in the S.S. to arrive to the desired result

$$v(y) = \frac{\delta + sk\overline{V}(y)}{\left(\delta + sk\overline{V}(y)\right) + (kU + skH_y(y))} \cdot n(y)$$

Once we have worked out a close form expression for the number of vacancies and the number of workers in y-type firms, we can deal with the number of unemployed and the number of workers with x-characteristic. From the differential equation for unemployed, substitute the balance condition for $h_x(x)$, such that

$$kVu(x) = \delta h_x(x) \Leftrightarrow kVu(x) = \delta\left(l(x) - u(x)\right) \Leftrightarrow \left(\delta + kV\right)u(x) = \delta l(x)$$
$$u(x) = \frac{\delta}{(\delta + kV)} l(x)$$

With this expression and basic algebra we work out $h_x(x)$

$$h_x(x) = \frac{kV}{(\delta + kV)} l(x)$$

Finally, we are ready to calculate $h(x, y)$. The steps to arrive at the solution are basically the same as those to compute $h_y(y)$. THen, consider the last expression and rearrange such that U-to-H and H-to-U flows are on the LHS and H-to-H flows are in the RHS.

$$\delta h(x,y) - kv(y)u(x) = sk\int_{\underline{y}}^{y} h(x,y')dy'v(y) - sk\int_{y}^{\overline{y}} v(y')dy'h(x,y)$$

Integrate both sites from $\underline{y}$ to y to have

$$\delta\int_{\underline{y}}^{y} h(x,y')dy' - ku(x)V(y) = -sk\int_{\underline{y}}^{y} h(x,y')dy'\overline{V}(y)$$

rearranging

$$\left(\delta + sk\overline{V}(y)\right)\int_{\underline{y}}^{y} h(x,y')dy' = ku(x)V(y)$$
$$\int_{\underline{y}}^{y} h(x,y')dy' = \frac{ku(x)V(y)}{\left(\delta + sk\overline{V}(y)\right)}$$

And deriving with respect to to y we arrive at the final form

$$h(x,y) = \frac{\delta + skV}{\left(\delta + sk\overline{V}(y)\right)^2} \cdot ku(x)v(y)$$

Which is difficult to interpret. However, we can prove that the following interesting result holds, $h(x,y) = \frac{1}{H}h_x(x)h_y(y)$. Start by plugging in $h(x,y)$ the expressions for $ku(x) = \frac{\delta}{V}h_x(x)$, $v(y)$, and solve for $\left(\delta + sk\overline{V}(y)\right)$ in equation 9, so that we get

$$h(x,y) = \frac{(\delta + skV)}{\left(\frac{kV(y)U}{H_y(y)}\right)^2} \cdot \frac{(\delta + skV)}{(kU + skH_y(y))^2} \cdot kUh_y(y)\frac{\delta}{V}h_x(x)$$

Solve for $(kU + skH_y(y))$ in 9 and use the fact that $kU = \frac{\delta}{V}H$

$$h(x,y) = \frac{(\delta + skV)}{\left(\frac{kV(y)U}{H_y(y)}\right)^2} \cdot \frac{(\delta + skV)}{\left(\frac{(\delta+skV)H_y(y)}{V(y)}\right)^2} \cdot \left(\frac{\delta}{V}\right)^2 Hh_x(x)h_y(y)$$

Cancelling out terms and substituting

$$h(x,y) = \left(\frac{\delta}{kUV}\right)^2 Hh_x(x)h_y(y) = \frac{1}{H^2}Hh_x(x)h_y(y) = \frac{1}{H}h_x(x)h_y(y)$$

Now, we are ready to calculate the distribution of wages from the flow equation

$$\begin{aligned}\frac{dG_t(w|x,y) \cdot h_t(x,y)}{dt} &= kv_t(y)u_t(x) + sk\int_{\underline{y}}^{q} h(x,y')dy' \cdot v(y) \\ &- \delta G_t(w|x,y) \cdot h_t(x,y) - sk\int_{q}^{\overline{y}} v(y')dy' G_t(w|x,y) \cdot h_t(x,y) = 0\end{aligned}$$

rearranging

$$\left(\delta + sk\int_{q}^{\overline{y}} v(y')dy'\right) G(w|x,y) \cdot h(x,y) = kv(y)u(x) + sk\int_{\underline{y}}^{q} h(x,y')dy'v(y)$$

solving for $G_t(w|x,y)$ we have

$$G(w|x,y) = \frac{\left(kv(y)u(x) + sk\int_{\underline{y}}^{q} h(x,y')dy'v(y)\right)}{\left(\delta + sk\int_{q}^{\overline{y}} v(y')dy'\right)} \cdot \frac{1}{h(x,y)}$$

Substitute $h(x,y)$ by the product of the marginals $\frac{1}{H}h_x(x)h_y(y)$. Also use the flow equation for the unemployed $kVu(x) = h_x(x)$ and the aggregate flow equation $kVU = \delta H$ to arrive at $u(x) = \frac{U}{H}h_x(x)$, then after cancelling terms

$$G(w|y) = \frac{\left(kU + sk\int_{\underline{y}}^{q} h_y(y')dy'\right)}{\left(\delta + sk\int_{q}^{\overline{y}} v(y')dy'\right)} \cdot \frac{v(y)}{h_y(y)}$$

Which shows what intuition could have told us in advance, namely that the distribution of wages does not depend on x.

D.2 Distributions with Minimum Wages

To find out the distributions of matches, vacancies and unemployed under the minimum wage, $\tilde{h}(x,y)$, $\tilde{v}(y)$ and $\tilde{u}(x)$ respectively, it will just suffice to rewrite them in terms of the old ones. Under the minimum wage meetings $xy < m$, which could have ended in a match, are not possible. Then the balance conditions can be rewritten as

$$l(x) = \underbrace{u(x) + \int_{\underline{y}}^{\hat{y}(x)} h(x,y')dy'}_{\tilde{u}(x)} + \underbrace{\int_{\hat{y}(x)}^{\overline{y}} h(x,y')dy'}_{\tilde{h}_x(x)}$$

$$n(y) = \underbrace{v(y) + \int_{\underline{x}}^{\hat{x}(y)} h(x',y)dx'}_{\tilde{v}(y)} + \underbrace{\int_{\hat{x}(y)}^{\overline{x}} h(x',y)dx'}_{\tilde{h}_y(y)}.$$

Because the previous analysis without minimum wages, we know that under random search there is no sorting under the model assumptions. The implication being to express $h(x,y)$ as the product of two functions that describe abilities and productivities independently, namely $h(x,y) = \frac{1}{H} h_y(y) h_x(x)$, then the balance conditions can be rewritten as

$$l(x) = \underbrace{u(x) + h_x(x) \int_{\underline{y}}^{\hat{y}(x)} \frac{h_y(y')}{H} dy'}_{\tilde{u}(x)} + \underbrace{h_x(x) \int_{\hat{y}(x)}^{\overline{y}} \frac{h_y(y')}{H} dy'}_{\tilde{h}_x(x)}$$

$$n(y) = \underbrace{v(y) + h_y(y) \int_{\underline{x}}^{\hat{x}(y)} \frac{h_x(x')}{H} dx'}_{\tilde{v}(y)} + \underbrace{h_y(y) \int_{\hat{x}(y)}^{\overline{x}} \frac{h_x(x')}{H} dx'}_{\tilde{h}_y(y)}$$

.

And then as

$$l(x) = \underbrace{u(x) + h_x(x) F_y(\hat{y}(x))}_{\tilde{u}(x)} + \underbrace{h_x(x) \overline{F}_y(\hat{y}(x))}_{\tilde{h}_x(x)}$$

$$n(y) = \underbrace{v(y) + h_y(y) F_x(\hat{x}(y))}_{\tilde{v}(y)} + \underbrace{h_y(y) \overline{F}_x(\hat{x}(y))}_{\tilde{h}_y(y)}.$$

Where $F_i(\hat{i}(j)) = \int_{\underline{i}}^{\hat{i}(j)} \frac{h_i(i')}{H} di'$ and $\overline{F}_i(\hat{i}(j)) = 1 - F_i(\hat{i}(j))$. Integrating over abilities in the first condition and over productivities in the second we work out the aggregate balance conditions

$$L = \underbrace{U + \int_{\underline{x}}^{\overline{x}} h_x(x') F_y(\hat{y}(x'))\, dx'}_{\tilde{U}} + \underbrace{\int_{\underline{x}}^{\overline{x}} h_x(x') \overline{F}_y(\hat{y}(x'))\, dx'}_{\tilde{H}}$$

$$N = \underbrace{V + \int_{\underline{y}}^{\overline{y}} h_y(y') F_x(\hat{x}(y'))\, dy'}_{\tilde{V}} + \underbrace{\int_{\underline{y}}^{\overline{y}} h_y(y') \overline{F}_x(\hat{x}(y'))\, dy'}_{\tilde{H}}.$$

All of the aggregates: $\tilde{H}$, $\tilde{U}$, and $\tilde{V}$, and the distributions: $h(x,y)$, $\tilde{u}(x)$, and $\tilde{v}(y)$, can be worked out from the follow equation for jobs:

$$0 = k\tilde{v}(y)\tilde{u}(x) + sk \int_{\hat{y}(x)}^{q} h(x,y') dy' \tilde{v}(y) - \delta \tilde{G}(w|x,y) \cdot h(x,y) - sk \int_{q}^{\overline{y}} \tilde{v}(y') dy' \tilde{G}(w|x,y) \cdot h(x,y). \tag{10}$$

To derive an expression for $\tilde{H}$, evaluate $G(w|x,y)$ at $w = xy$ so $G(xy|x,yy) = 1$, and integrate 10 over all x and y. Notice that in the aggregate all the workers that quit are the same as those who are poached, i.e. $\int_{\hat{y}(x)}^{y} h_t(x,y')dy'v_t(y) = \int_{y}^{\overline{y}} v_t(y')dy'h_t(x,y)$, hence

$$k\tilde{V}\tilde{U} - \delta\tilde{H} = 0$$

Then using the aggregate balance equations under the minimum wage, and following the same steps as in the previous section, we have an expression for $\tilde{H}$, and reusing the balance equations we have functional forms for $\tilde{V}$ and $\tilde{U}$.

To derive forms of $\tilde{u}(x)$ and $\tilde{h}(x)$, evaluate $G(.|xy)$ at $w = xy$ in 10, integrate over all y, and remember that quits equal poaches, then

$$k\tilde{V}\tilde{u}(x) = \delta\tilde{h}(x).$$

And the usual expressions follow

$$\tilde{u}(x) = \frac{\delta}{\delta + k\tilde{V}} l(x) \quad \text{and} \quad \tilde{h}(x) = \frac{k\tilde{V}}{\delta + k\tilde{V}} l(x).$$

To work out expressions for $v(y)$ and $h_y(y)$, evaluate $G(.|xy)$ at $w = xy$ in 10, and integrate over all x, observing that to have

$$\begin{aligned} k\tilde{v}(y)\tilde{U} - \delta \int_{\underline{x}}^{\overline{x}} h(x',y) 1\{x'y \geq m\}\, dx' \\ = sk \int_{y}^{\overline{y}} \tilde{v}(y')dy' \cdot \int_{\underline{x}}^{\overline{x}} h(x',y) 1\{x'y \geq m\}\, dx' - sk \int_{\underline{x}}^{\overline{x}} \int_{\frac{m}{x}}^{y} h(x',y')dy'dx' \cdot \tilde{v}(y). \end{aligned}$$

Where the indicator function $1\{.\}$ takes the value of 1 if a condition is met. Now, change the order of integration in the last term, taking care of the limits, and use the definition $\tilde{h}_y(y) = h_y(y) \int_{\hat{x}(y)}^{\overline{x}} \frac{h_x(x')}{H} dx'$ given in the balance condition to arrive at

$$k\tilde{v}(y)\tilde{U} - \delta\tilde{h}_y(y) = sk \int_{y}^{\overline{y}} \tilde{v}(y')dy' \cdot \tilde{h}_y(y) - sk \int_{\underline{y}}^{y} \tilde{h}_y(y)dy' \cdot \tilde{v}(y).$$

Following the same steps as in the previous section, we can write:

$$\tilde{v}(y) = \frac{\delta + sk\tilde{\overline{V}}(y)}{\left(\delta + sk\tilde{\overline{V}}(y)\right) + \left(k\tilde{U} + sk\tilde{H}_y(y)\right)} \cdot n(y)$$

and

$$\tilde{h}_y(y) = \frac{k\tilde{U} + sk\tilde{H}_y(y)}{\left(\delta + sk\tilde{\overline{V}}(y)\right) + \left(k\tilde{U} + sk\tilde{H}_y(y)\right)} \cdot n(y)$$

Finally, we can derive expressions for $h(x, y)$ and $G(w|x, y)$. For $h(x, y)$, evaluate 10 at $w = xy$, integrate over $y' \in [\hat{y}(x), y]$, solve for $\int_{\hat{y}(x)}^{y} h(x, y')dy'$, and derive both sides with respect to y to arrive at

$$h(x, y) = \frac{\delta + sk\tilde{V}}{\left(\delta + sk\tilde{\overline{V}}(y)\right)^2} \cdot k\tilde{u}(x)\tilde{v}(y) \tag{11}$$

In the spirit of the preceding section, plugging the expressions $ku(x) = \frac{\delta}{V}h_x(x)$, $v(y)$, and $\left(\delta + sk\overline{V}(y)\right)$ in equation 11, we arrive at

$$h(x, y) = \begin{cases} \frac{1}{\tilde{H}}\tilde{h}_y(y)\tilde{h}_x(x) & if \quad xy \geq m^* \\ 0 & if \quad xy < m^* \end{cases}$$

The distributions are still independent because search is undirected. But, sorting is introduced by precluding matches of low-ability workers with low-productivity firms. Again, look at the section above for the distributions of wages $\tilde{G}(w|y)$

$$\tilde{G}(w|y) = \begin{cases} \frac{\tilde{h}_y(q)}{\tilde{v}_y(q)} \cdot \frac{\tilde{v}_y(y)}{\tilde{h}_y(y)} & if \quad xy \geq m^* \\ G_0 \quad if & xy = m^* \text{ or } \underline{y} = b \\ 0 & if \quad xy < m^* \end{cases}$$

E Estimation

I estimate the model using the classical minimum distance (CMD) estimation. Excellent references for reviewing this area are Newey and McFadden (1994) and Wooldridge (2010), which are followed in this paper. For this purpose moments are taken from data and stored in a vector $\widehat{\mathbf{m}}$ of size K. The k^{th}-element is $\widehat{m_k} = \frac{1}{N}\sum_{i=1}^{N} m_{ki}$, where $\widehat{m_k}$ might be any statistic of interest like mean duration of unemployment. Then, the same set of moments is calculated from the model, which is numerically solved, given a set of parameters $\boldsymbol{\theta} = \left\{\underline{x}, \sigma_x, s, \eta, \underline{y}, \gamma_{sh}, \gamma_{sc}, c_0, c_1\right\}$. Assuming that $\widehat{\mathbf{m}} \xrightarrow{p} \mathbf{m_0}$, there is a vector of functions such that $m_{k0} = m_k(\boldsymbol{\theta_0})$, where $m_k(\boldsymbol{\theta_0})$ is calculated from the model given a parameterization $\boldsymbol{\theta_0}$ which is meant to be unique. The objective is to make these theoretical moments as close as possible as their data counterparts. Then, the problem lays on retrieving the parameters that make the loss function as close to zero as possible, or in other words

$$\widehat{\boldsymbol{\theta}} = \underset{\boldsymbol{\theta}}{\text{argmin}}\left\{\left(\widehat{m} - m(\boldsymbol{\theta})\right)' W\left(\widehat{m} - m\left(\boldsymbol{\theta}\right)\right)\right\}.$$

Where W is a positive semi-definite weighting matrix, although in the econometric estimation it is just a diagonal matrix with the inverse of variance of their respective moments (Flinn and Mullins, 2019).

Once the programme has been set, it is only to choose the right moments to match in order to retrieve the underlying parameters. Since my model introduces free entry of firms to the canonical Postel-Vinay and Robin (2002), it is difficult to argue what moments identify what parameters since moments are affected by all parameters of the model in some way. The approach taken here is the one of Lise and Robin (2017), where they use an heuristic approach to identify all the parameters at once, making sense of the sensitivity of some moments with respect to the parameters that they mean to identify. The mapping of what moments identify the parameters of the model are laid out in the main text.